

**Alone we can do so little;
together we can do so much.**

Helen Keller



Circuit theory Notes

**Notes By: Muhammad
Naeem Akhter**

fb/mna4uonly

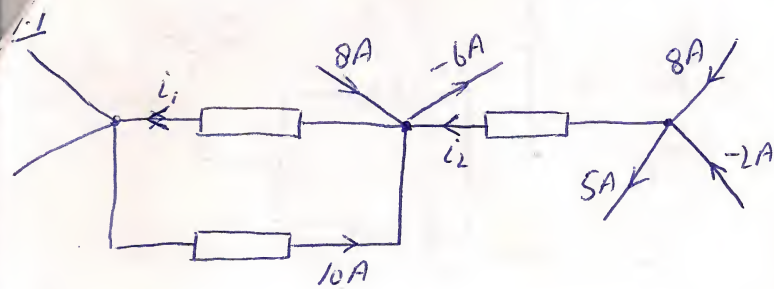
NOTES BY MUHAMAD NAEEM AKHTER. 2014/15

010FZ Circuit Theory

Academic Year 2014-2015

Prof. Stefano Grivet Talocia

F-100
F-100



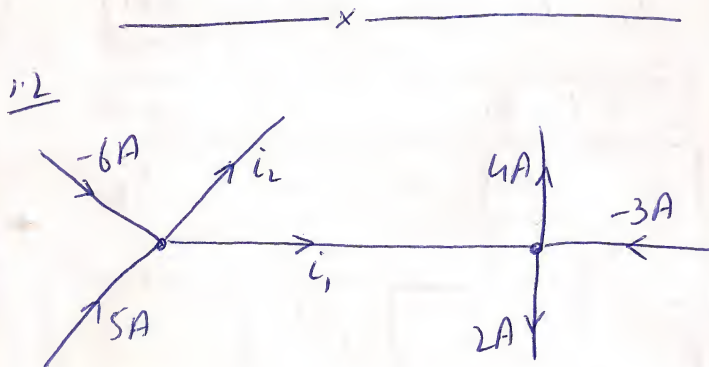
SOLUTION:-

$$1) -i_2 + 8 - 2 - 5 = 0$$

$$-i_2 + 1 = 0 \Rightarrow \boxed{i_2 = 1A}$$

$$2) -i_1 + 1 + 8 + 6 + 10 = 0$$

$$-i_1 + 25 = 0 \Rightarrow \boxed{i_1 = 25A}$$



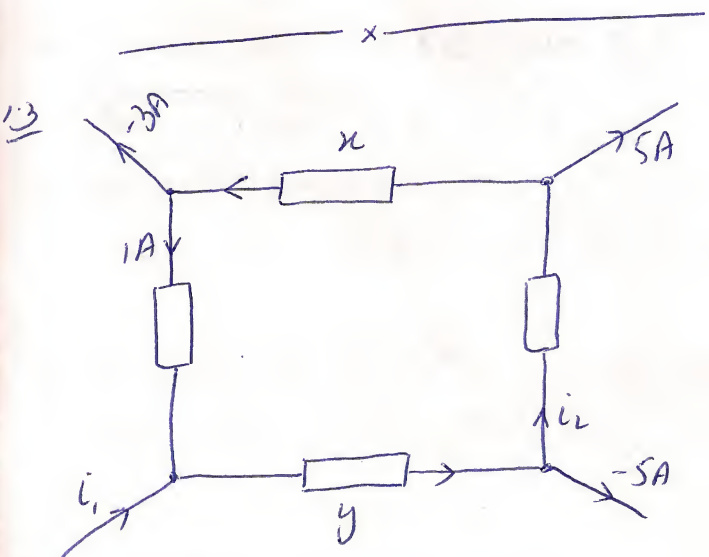
SOLUTION:-

$$1) i_1 - 4 - 3 - 2 = 0$$

$$i_1 - 9 = 0 \Rightarrow \boxed{i_1 = 9A}$$

$$2) -i_2 - 6 + 5 - 9 = 0$$

$$-i_2 - 10 = 0 \Rightarrow \boxed{i_2 = -10A}$$



$$i_4 + 3 - 1 = 0 \Rightarrow i_4 = -2A$$

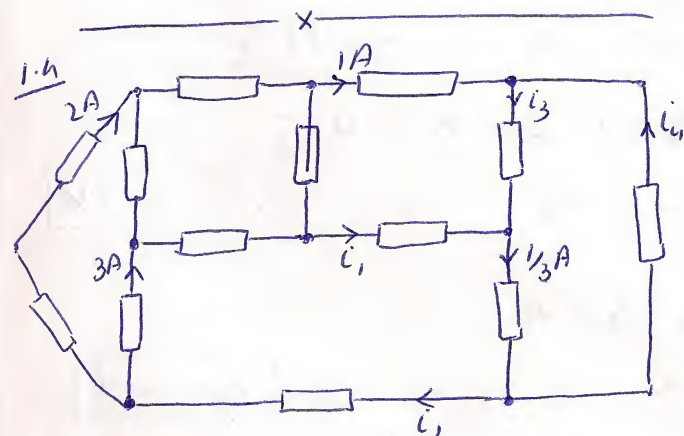
$$1) i_2 - 5 + 2 = 0$$

$$i_2 - 3 = 0 \Rightarrow \boxed{i_2 = 3A}$$

$$i_3 + 5 - 3 = 0 \Rightarrow i_3 = -2A$$

$$2) i_1 + 2 + 1 = 0$$

$$i_1 + 3 = 0 \Rightarrow \boxed{i_1 = -3A}$$



$$1) i_2 - 2 - 3 = 0$$

$$i_2 - 5 = 0 \Rightarrow \boxed{i_2 = 5A}$$

$$2) -5 - i_4 + \frac{1}{3} = 0$$

$$-i_4 - \frac{14}{3} = 0 \Rightarrow \boxed{i_4 = -\frac{14}{3}A}$$

$$3) -i_3 + 1 - \frac{14}{3} = 0$$

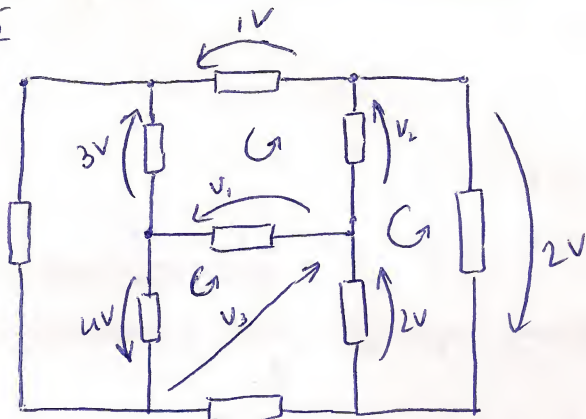
$$-i_3 - \frac{11}{3} = 0 \Rightarrow \boxed{i_3 = -\frac{11}{3}A}$$

$$4) i_1 - \frac{11}{3} - \frac{1}{3} = 0$$

$$i_1 - \frac{12}{3} = 0$$

$$\boxed{i_1 = 4A}$$

1.5



SOLUTION-

$$1) -v_2 - 2 - 2 = 0$$

$$-v_2 - 4 = 0 \Rightarrow \boxed{v_2 = -4V}$$

$$2) -4 + 1 - 3 - v_1 = 0$$

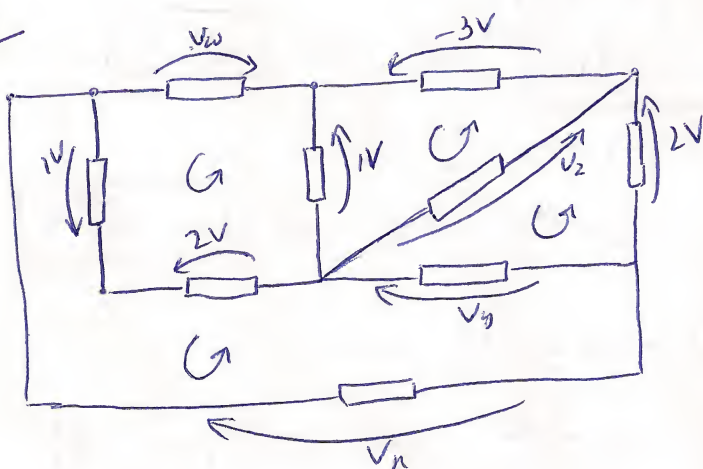
$$-v_1 - 6 = 0 \Rightarrow \boxed{v_1 = -6V}$$

$$3) v_3 - 6 + 4 = 0$$

$$v_3 - 2 = 0 \Rightarrow \boxed{v_3 = 2V}$$

b)

1.6



$$\text{SOLUTION- } 1) -v_w + 1 - 2 + 1 = 0$$

$$\boxed{v_w = 0V}$$

$$2) v_z - 3 - 1 = 0$$

$$v_z - 4 = 0 \Rightarrow \boxed{v_z = 4V}$$

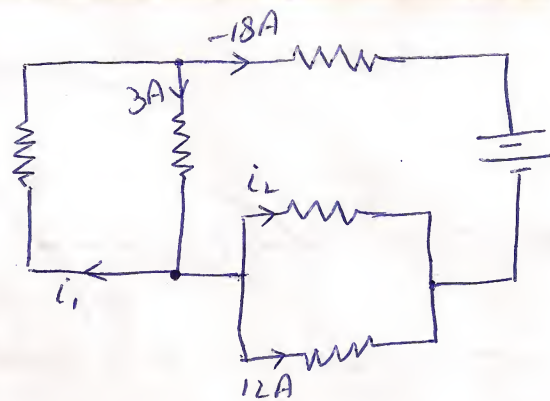
$$3) -v_y + 2 - 4 = 0$$

$$-v_y - 2 = 0 \Rightarrow \boxed{v_y = -2V}$$

$$4) -v_n + 2 - 3 = 0$$

$$-v_n - 1 = 0 \Rightarrow \boxed{v_n = -1V}$$

1.7



$$\text{SOLUTION- } 1) i_1 - 3 + 18 = 0$$

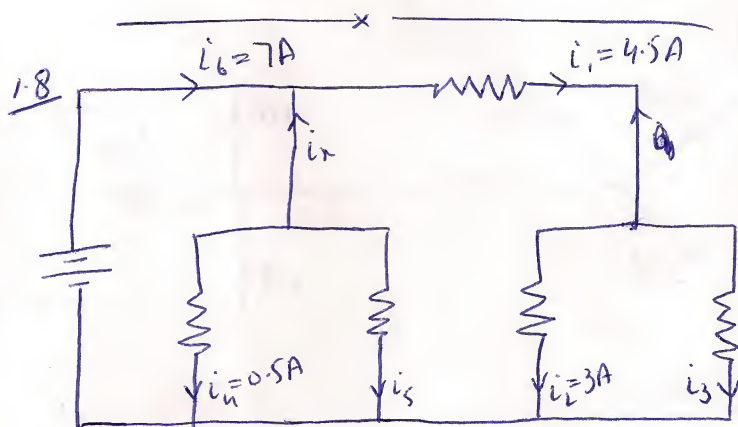
$$i_1 + 15 = 0 \Rightarrow \boxed{i_1 = -15A}$$

$$2) i_2 + 3 + 15 = 0$$

$$-i_2 + 18 = 0 \Rightarrow i_2$$

$$i_2 + 12 - 18 = 0$$

$$i_2 - 6 = 0 \Rightarrow \boxed{i_2 = 6A}$$



$$1) i_n + 7 - 4.5 = 0$$

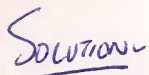
$$i_n + 2.5 = 0 \Rightarrow i_n = -2.5A$$

$$-i_5 + 2.5 - 0.5 = 0$$

$$-i_5 + 2 = 0 \Rightarrow \boxed{i_5 = 2A}$$

$$2) -i_3 - 3 + 4.5 = 0$$

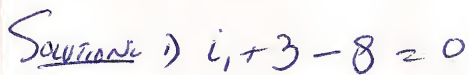
$$-i_3 + 1.5 = 0 \Rightarrow \boxed{i_3 = 1.5A}$$



$$V_{ad} + 3 = 0 \Rightarrow V_{ad} = -3V$$

$$V_1 - 9 \geq 0 \Rightarrow \boxed{V_1 \geq 9V}$$

$$V_{bc} - 4 = 0 \Rightarrow \boxed{V_{bc} = 4V}$$



$$i_1 - 5 = 0 \Rightarrow i_1 = 5A$$

$$i_L - 2 = 0 \Rightarrow i_L = 2A$$

$$V_{ad} = 7 \text{ V} \Rightarrow \boxed{V_{ad} = 7 \text{ V}}$$

$$V_n - 12 = 0 \quad \Rightarrow \quad V_n = 12V$$



$$i_L = -1A$$

$V_1 = -2V$

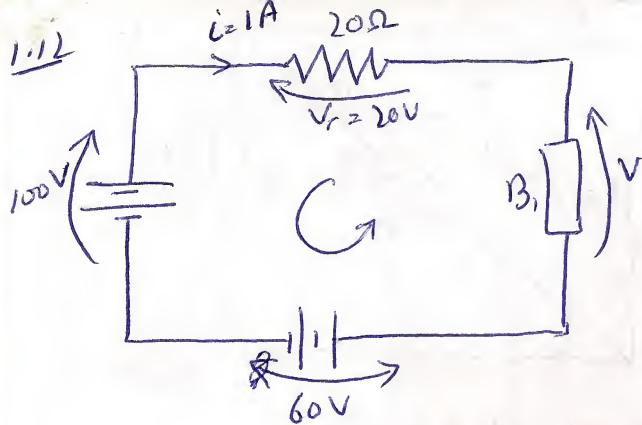
$$V_3 = 4V$$

$$1) P_1 = V_1 i_1$$
$$= 2(-2) = -4W$$

$$2) P_L = V_L i_L = 4(-1) = -4W$$

$$3) P_3 = V_3 i_3 = 4(3) = 12W$$

$$h) P_h = V_h i_h$$
$$= 2 \text{ (V)} = -4 \text{ W}$$



As $P = Vi$
 $i = \frac{100}{100} = 1A \Rightarrow \boxed{i_2 = 1A}$

$V_r = 20(1) = 20V$

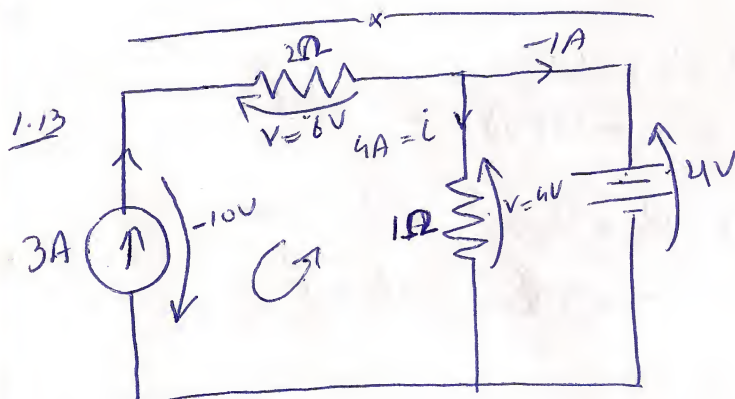
$V + 20 - 100 + 60 = 0$

$V - 20 = 0$

$\boxed{V = 20V}$

$P_r = V_r i$

$P_r = 20(1) \Rightarrow \boxed{P_r = 20W}$



$i_2 = 4A$

$V_r = 6V$

$V_I = 4 + 6 = 0$

$V_I = -10V$

$P(E = 4V) = 4(-1) = -4W$

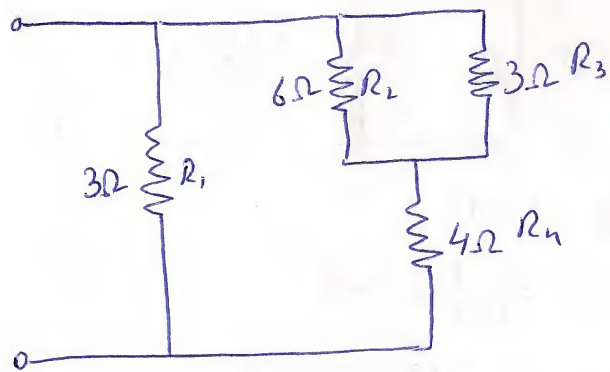
$P(R = 1\Omega) = 4(4) = 16W$

$P(R = 2\Omega) = 6(3) = 18W$

$P(I = 3A) = -10(3) = -30W$

CHAPTER-2 RESISTIVE CIRCUITS

compute resistance from A & B.

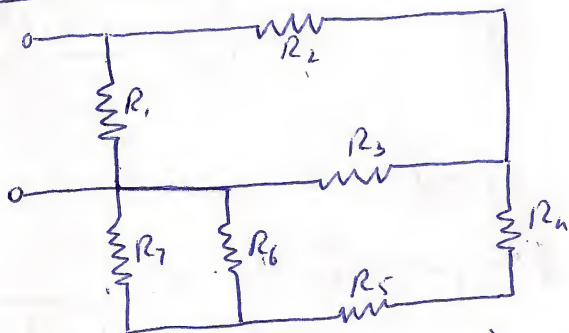


SOLUTION-

$$\begin{aligned} & ((R_2 \parallel R_3) + R_4) \parallel R_1 \\ &= \left(\frac{6 \cdot 3}{6+3} + 4 \right) \parallel R_1 \\ &= \left(\frac{18}{9} + 4 \right) \parallel R_1 \\ &= \frac{6 \cdot 3}{6+3} = \frac{18}{9} = 2 \Omega \end{aligned}$$

$$R_{eq} = 2 \Omega \text{ Ans}$$

2.2

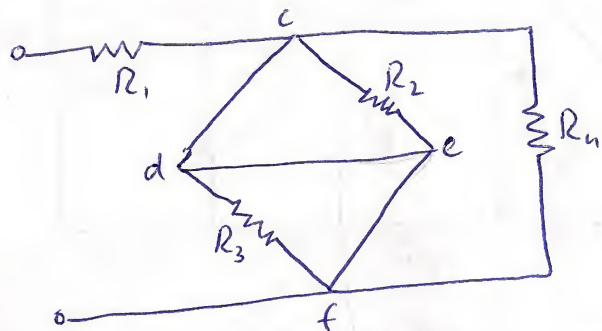


$$\begin{aligned} & ((R_2 \parallel R_6 + R_5 + R_4) \parallel R_3 + R_1) \parallel R_1 \\ & R_{eq} = \left(\frac{1000 \cdot 1000}{2000} + 1000 + 1000 \parallel R_3 + R_1 \right) \parallel R_1 \end{aligned}$$

$$\begin{aligned} R_{eq} &= \frac{2500 \cdot 1000}{3500} = \frac{25000}{35} \\ &= (714.285 + 1000) \parallel R_1 \\ &= \frac{1714.285 \times 1000}{2714.285} \end{aligned}$$

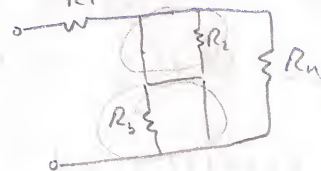
$$R_{eq} = 631.57 \Omega$$

2.3



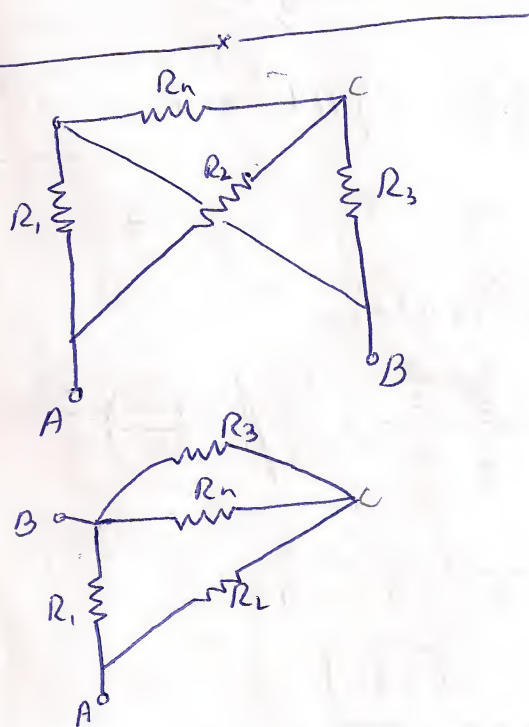
$$R_{eq} = R_1$$

$$R_{eq} = 15 \Omega$$



$$R_{eq} = R_1$$

2.4



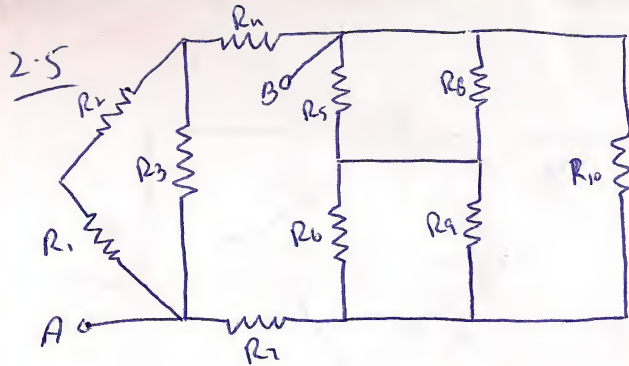
$$R_{eq} = (R_3 \parallel R_4 + R_2) \parallel R_1$$

$$R_{eq} = \left(\frac{40 \cdot 40}{80} + 20 \right) \parallel R_1$$

$$R_{eq} = (20 + 20) \parallel R_1$$

$$= \frac{40 \cdot 10}{50}$$

$$R_{eq} = 8 \Omega$$



$$1) R_{eq}' = (R_1 + R_2) \parallel R_3 + R_4$$

$$R_{eq}'' = (R_5 \parallel R_8 + R_6 \parallel R_9) \parallel R_{10} + R_7$$

$$R_{eq}' = (1+1) \parallel 1 + 1$$

$$= \frac{2 \cdot 1}{2+1} + 1$$

$$= \frac{2}{3} + 1 = \frac{5}{3} \Omega$$

$$R_{eq}'' = \left(\frac{1}{2} + \frac{1}{2}\right) \parallel 1 + 1$$

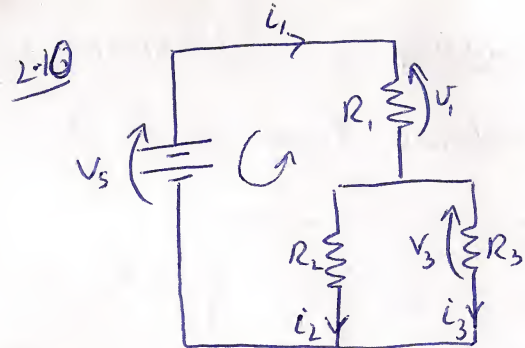
$$= \frac{1}{2} + 1 = \frac{3}{2} \Omega$$

$$R_{eq} = R_{eq}' \parallel R_{eq}''$$

$$= \frac{\frac{5}{3} \cdot \frac{3}{2}}{\frac{5}{3} + \frac{3}{2}} = \frac{5}{2} \left(\frac{10+9}{6} \right)$$

$$= \frac{5}{2} \times \frac{6^3}{19} = \frac{15}{19}$$

$$R_{eq} = 0.789 \Omega$$



$$R_{eq} = R_2 \parallel R_3 + R_1$$

$$= \frac{30 \cdot 70}{100} + 19$$

$$R_{eq} = 21 + 19 = 40 \Omega$$

$$V_s = i_1 R_{eq}$$

$$i_1 = \frac{60}{40} \Rightarrow i_1 = 1.5 A$$

$$V_1 = i_1 R_1$$

$$V_1 = (1.5) 19 \Rightarrow V_1 = 28.5 V$$

$$-V_s + V_3 + V_1 = 0$$

$$-60 + V_3 + 28.5 = 0 \Rightarrow V_3 = 31.5 V$$

$$V_3 = i_3 R_3$$

$$i_3 = \frac{31.5}{70} \Rightarrow i_3 = 0.45 A$$

$$i_1 - i_2 - i_3 = 0$$

$$1.5 - i_2 - 0.45 = 0$$

$$i_2 = 1.5 - 0.45 \Rightarrow i_2 = 1.05 A$$

$$V_1 = \frac{19}{40} \times 60 = 28.5 V$$

$$V_3 = \frac{21}{40} \times 60 = 31.5 V$$

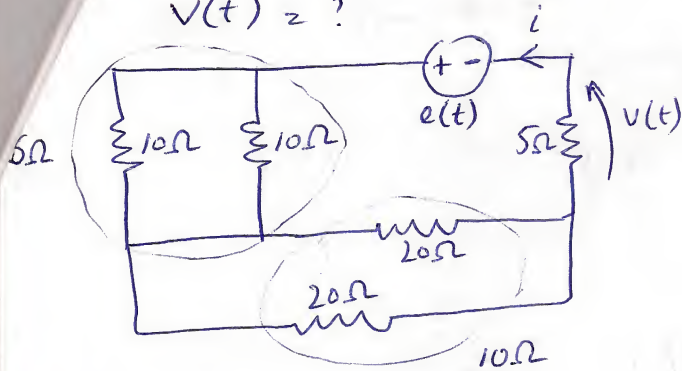
$$V_s = i_1 R_{eq} \Rightarrow i_1 = 1.5 A$$

$$i_2 = \frac{70}{100} \times 1.5 \Rightarrow i_2 = 1.05 A$$

$$i_3 = \frac{30}{100} \times 1.5 \Rightarrow i_3 = 0.45 A$$

$$e(t) = E_0 \sin(\omega t + \phi)$$

$$v(t) = ?$$



$$R_{eq} = 5 + 10 + 5 = 20 \Omega$$

$$i = \frac{e(t)}{R_{eq}}$$

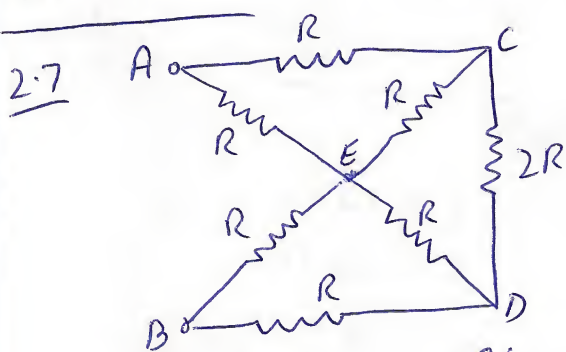
$$i = \frac{E_0 \sin(\omega t + \phi)}{20}$$

$$v(t) = i \cdot R$$

$$= \frac{5 E_0 \sin(\omega t + \phi)}{20}$$

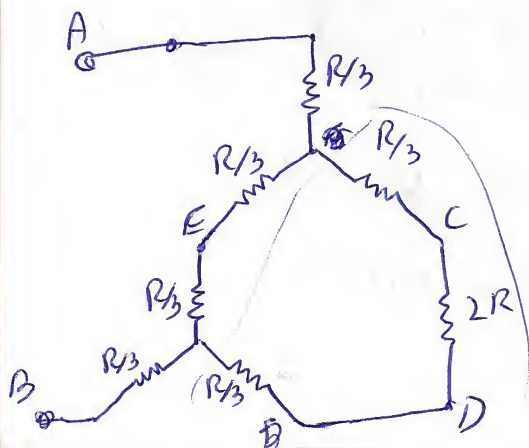
$$v(t) = -0.25 E_0 \sin(\omega t + \phi)$$

(-ve) is due to the same direction of i & $v(t)$.



Transform from delta to star.

$$r = \frac{R}{3}$$



$$R_{eq} = \frac{R}{3} + \frac{R}{3} + \left[\left(\frac{R}{3} + \frac{R}{3} \right) \parallel \left(\frac{R}{3} + 2R + \frac{R}{3} \right) \right]$$

$$= \frac{2R}{3} + \left(\frac{2R}{3} \parallel \frac{2R}{3} + 2R \right)$$

$$= \frac{2(9)}{3} + \left(\frac{2(9)}{3} \parallel \frac{2(9)}{3} + 2(9) \right)$$

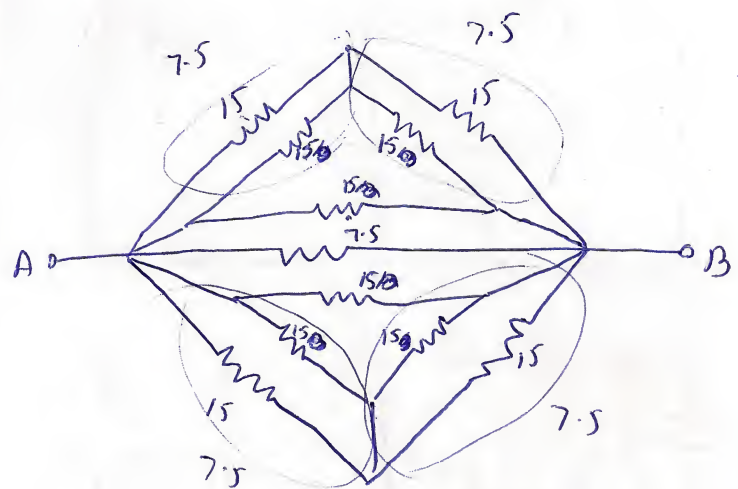
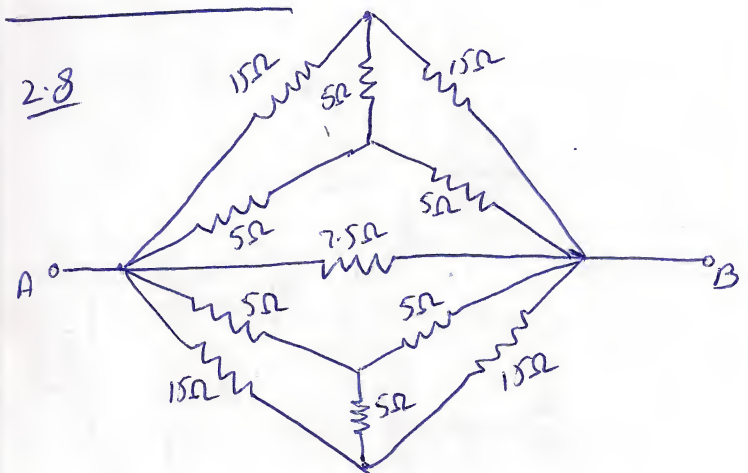
$$= 6 + (6 \parallel 24)$$

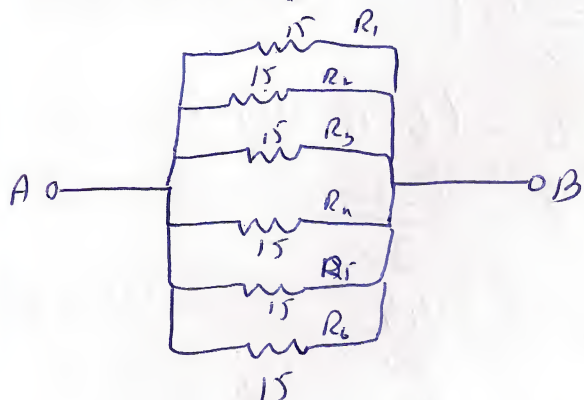
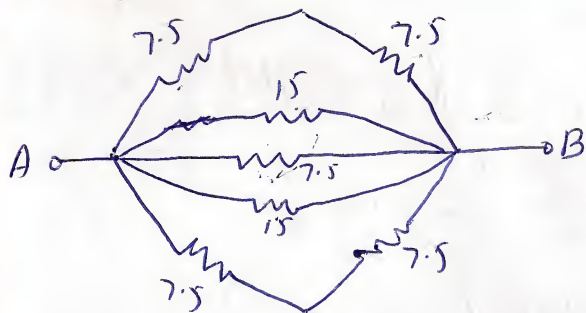
$$= 6 + \frac{6 \cdot 24}{30}$$

$$= 6 + \frac{144}{30}$$

$$= \frac{180 + 144}{30} = \frac{324}{30}$$

$$R_{eq} = 10.8 \Omega$$



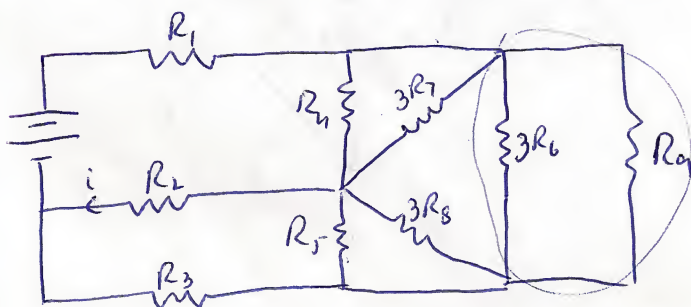
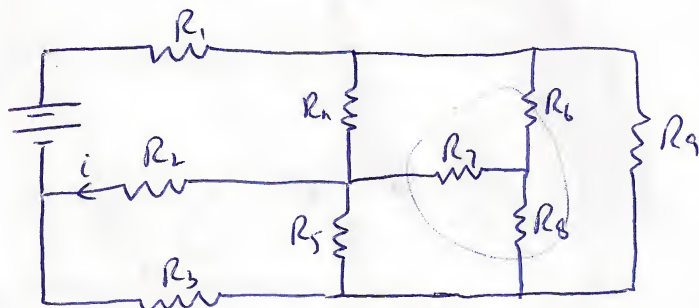
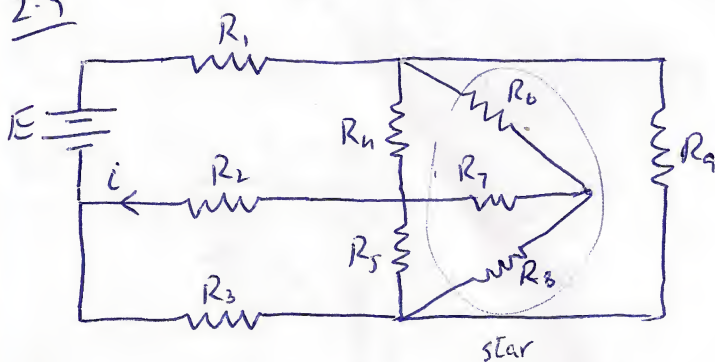


$$R_{eq} = \frac{R}{6}$$

$$= \frac{15}{6} = 2.5 \Omega$$

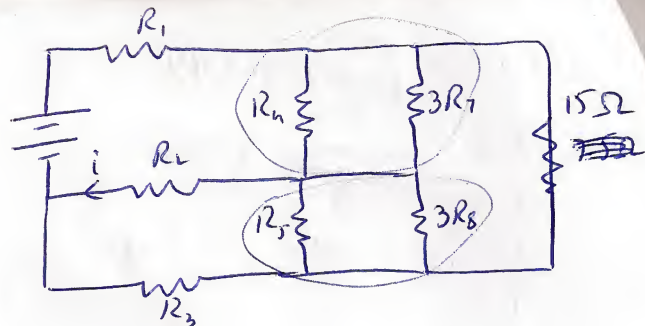
($R_1 = R_2 = R_3 = R_4 = R_5 = R_6$)

2.9



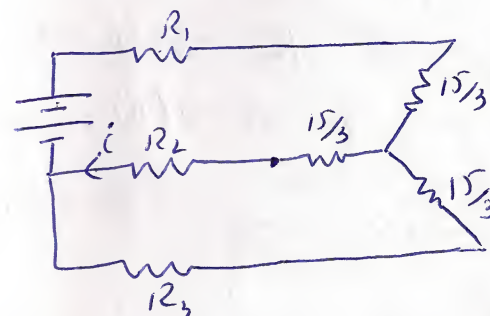
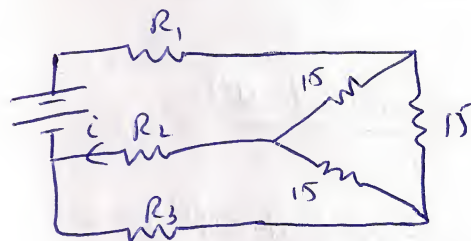
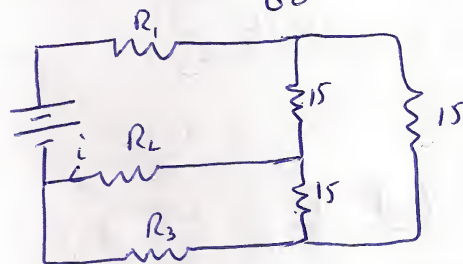
$$\frac{30 \cdot 30}{60} = \frac{900}{60} = \frac{30}{2} = 15$$

$$\frac{10 \cdot 30}{40} = \frac{300}{40} = 7.5 \Omega$$



$$R_4 || 3R_7 = \frac{30 \cdot 30}{60} = 15 \Omega$$

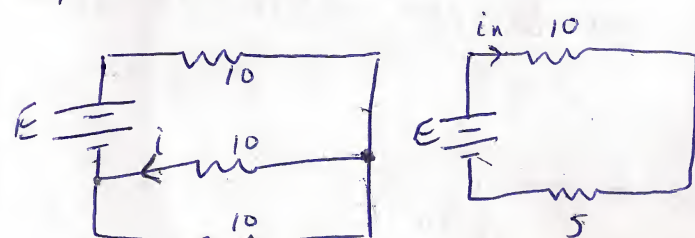
$$R_5 || 3R_8 = \frac{30 \cdot 30}{60} = 15 \Omega$$



$$R_3 + 5 = 5 + 5 = 10 \Omega$$

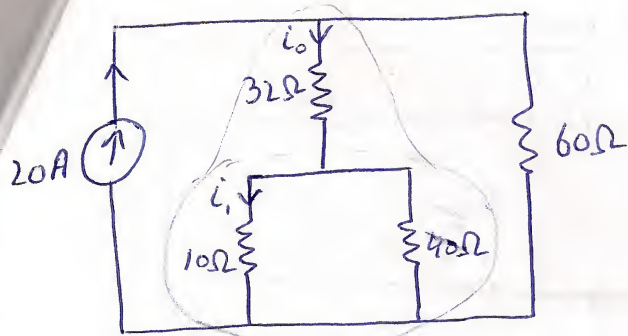
$$R_2 + 5 = 5 + 5 = 10 \Omega$$

$$R_1 + 5 = 5 + 5 = 10 \Omega$$



$$i_k = \frac{300}{10+5} \Rightarrow i_k = 20 A$$

$$\text{so } i(R=10\Omega) = 10 A$$



$$R_{eq} = 8\Omega + 32 = 40\Omega$$

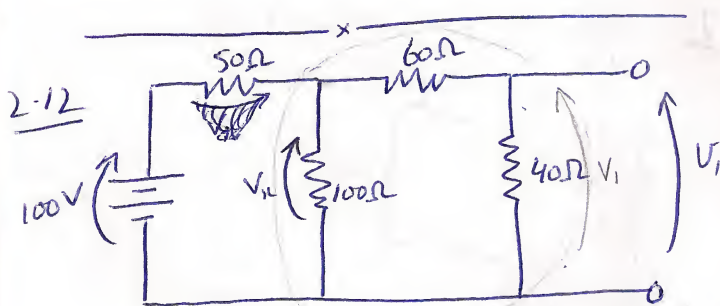
$$i_0 = \frac{60}{40+60} \times 20$$

$$= \frac{1200}{100} \Rightarrow i_0 = 12A$$

Now $i_1 = \frac{40}{10+40} \times 12$

$$= \frac{48}{5} \times 12$$

$$i_1 = \frac{48}{5} \Rightarrow \boxed{i_1 = 9.6A}$$



$$R_{eq} = 60 + 40 \parallel 100$$

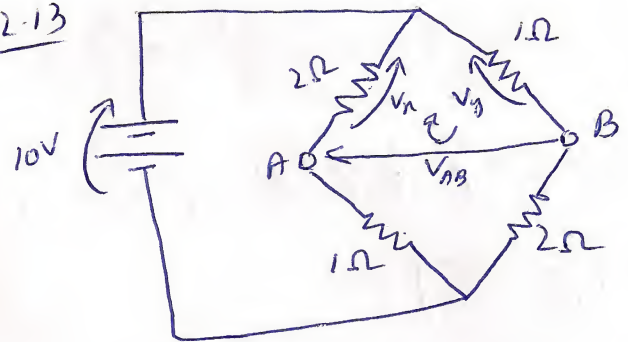
$$R_{eq} = \frac{100 \cdot 100}{200} = 50\Omega$$

$$V_x = \frac{50}{50+50} \times 100 = 50V$$

$$V_1 = \frac{40}{60+40} \times 50$$

$$= \frac{2000}{100} \Rightarrow \boxed{V_1 = 20V}$$

2.13



$$V_n = \frac{2}{2+1} \times 10$$

$$V_n = \frac{2}{3} \times 10 \Rightarrow V_n = 6.66V$$

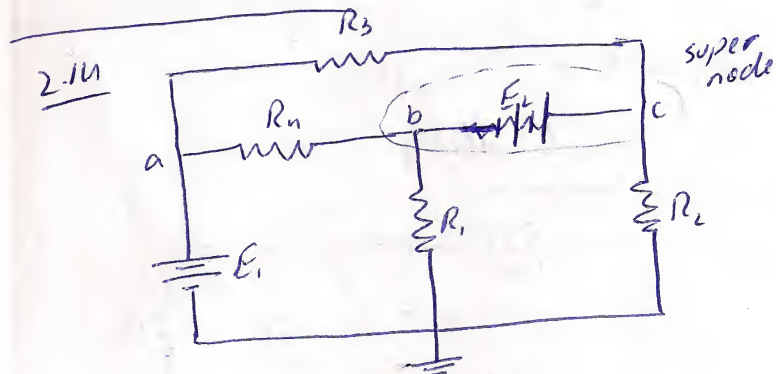
$$V_y = \frac{1}{2+1} \times 10$$

$$= \frac{1}{3} \times 10 \Rightarrow V_y = 3.33V$$

$$V_{AB} + V_n - V_y = 0$$

$$V_{AB} + 6.66 - 3.33 = 0$$

$$\boxed{V_{AB} = -3.33V}$$



$$V_a = E_1$$

$$V_c = E_2 + V_b$$

Taking super node apply KCL

$$\frac{V_b - V_a}{R_n} + \frac{V_c - V_b}{R_3} + \frac{V_b}{R_1} + \frac{V_c}{R_2} = 0$$

$$\frac{V_b - E_1}{R_n} + \frac{E_2 + V_b - E_1}{R_3} + \frac{V_b}{R_1} + \frac{E_2 + V_b}{R_2} = 0$$

$$\frac{V_b - E_1}{R_1} + \frac{V_b + E_2 - E_1}{R_3} + \frac{V_b + E_2}{R_2} + \frac{V_b}{R_4} = 0$$

$$V_b \left(\frac{1}{R_1} + \frac{1}{R_3} + \frac{1}{R_2} + \frac{1}{R_4} \right) + E_2 \left(\frac{1}{R_3} + \frac{1}{R_2} \right) - E_1 \left(\frac{1}{R_1} + \frac{1}{R_3} \right) = 0$$

$$V_b \left(\frac{1}{2} + \frac{1}{8} + \frac{1}{8} + \frac{1}{12} \right) + 10 \left(\frac{1}{8} + \frac{1}{8} \right) - 12 \left(\frac{1}{2} + \frac{1}{8} \right) = 0$$

$$V_b \left(\frac{9+1}{12} \right) + 10 \left(\frac{1}{4} \right) - 12 \left(\frac{5}{8} \right) = 0$$

$$V_b \left(\frac{10}{12} \right) + \frac{5}{2} - \frac{15}{2} = 0$$

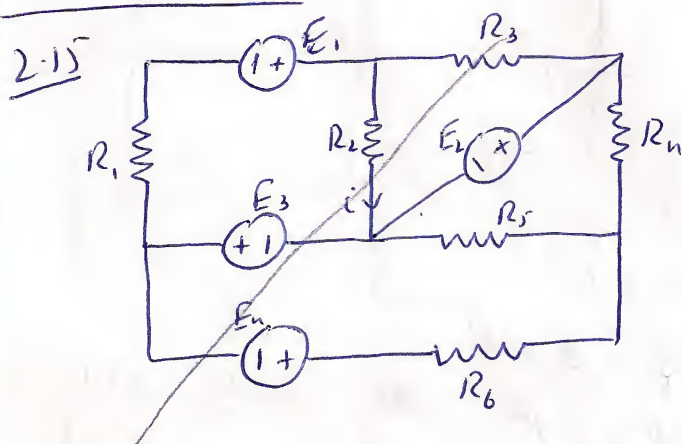
$$V_b \left(\frac{5}{6} \right) + \frac{5-15}{2} = 0$$

$$V_b \frac{5}{6} = \frac{10}{2}$$

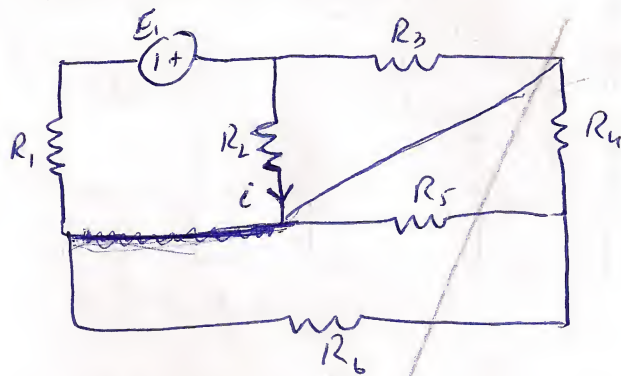
$$V_b = 5 \times \frac{6}{5}$$

$$V_b = 6V$$

$$\begin{aligned} V_a &= 12V \\ V_b &= 6V \\ V_c &= 10 + 6 = 16V \end{aligned} \quad \text{Ans}$$

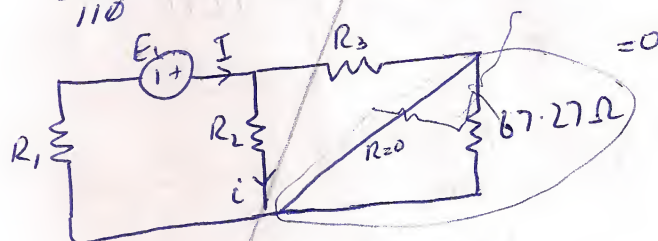


Contribution of E_1



$$R_5 || R_6 + R_4$$

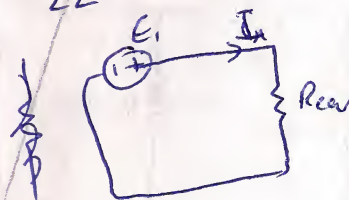
$$\frac{50 \cdot 60}{110} + 40 = 67.27 \Omega$$



$$R_2 || R_3 + R_1$$

$$R_{eq} = \frac{20 \cdot 30}{50} + 10 = 12 + 10 = 22 \Omega$$

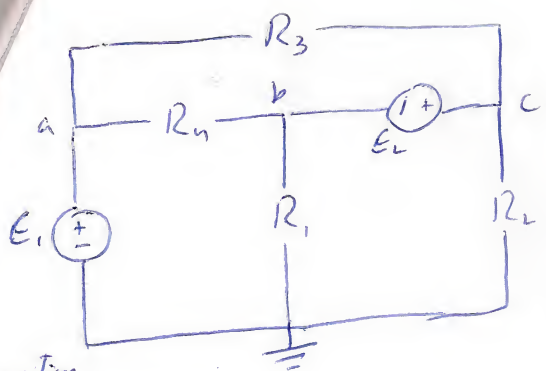
$$I_n = \frac{10}{22} \Rightarrow i_n = 0.45A$$



$$i_1 = \frac{R_3}{R_2 + R_3} I$$

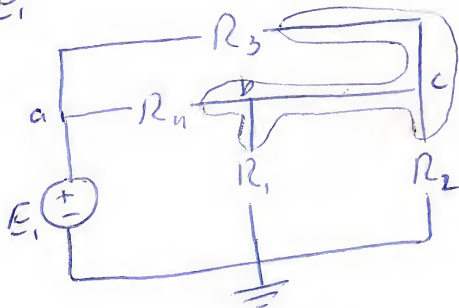
$$= \frac{30}{50} \times 0.45$$

$$i_1 = 0.27A$$



Superposition

$-E_1$



$$\dot{V}_a = E_1 \quad \dot{V}_b = \dot{V}_c$$

$$\frac{\dot{V}_b - E_1}{R_n} + \frac{\dot{V}_b}{R_1} + \frac{\dot{V}_b}{R_2} + \frac{\dot{V}_b - E_1}{R_3} = 0$$

$$\dot{V}_b \left(\frac{1}{2} + \frac{1}{12} + \frac{1}{8} + \frac{1}{8} \right) - \frac{12}{2} = 0$$

$$\dot{V}_b \left(\frac{24 + 4 + 6 + 6}{48} \right) = 6 + \frac{3}{2}$$

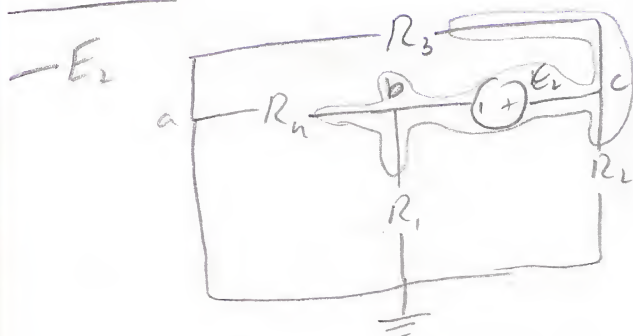
$$\dot{V}_b \left(\frac{40}{48} \right) = \frac{15}{2}$$

$$\dot{V}_b = \frac{6 \times 48}{40} \times \frac{15}{2}$$

$$\dot{V}_b = 9V$$

$$\dot{V}_a = 12V$$

$$\dot{V}_b = \dot{V}_c = 9V$$



$$\ddot{V}_a = 0$$

$$\ddot{V}_c = \ddot{V}_b + E_2$$

$$\frac{\ddot{V}_b}{R_n} + \frac{\ddot{V}_b}{R_1} + \frac{\ddot{V}_b + 10}{R_2} + \frac{\ddot{V}_b + 10}{R_3} = 0$$

$$\ddot{V}_b \left(\frac{1}{2} + \frac{1}{12} + \frac{1}{8} + \frac{1}{8} \right) + \frac{10}{8} + \frac{10}{8} = 0$$

$$\ddot{V}_b \left(\frac{40}{48} \right) = -\frac{5}{2}$$

$$\ddot{V}_b = -\frac{5}{2} \times \frac{48}{40}$$

$$\ddot{V}_b = -3V$$

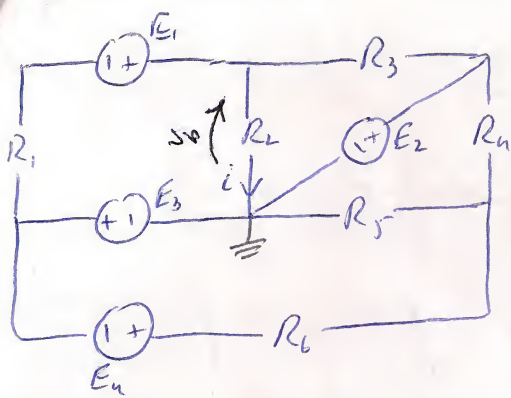
$$\ddot{V}_a = 0$$

$$\ddot{V}_b = -3V$$

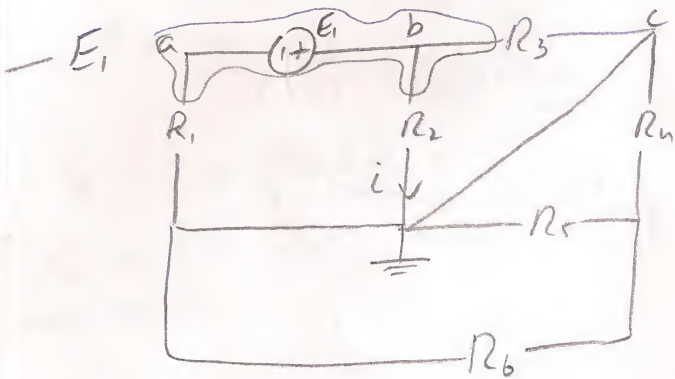
$$\ddot{V}_c = 7V$$

Hence

$$\left. \begin{aligned} V_a &= \dot{V}_a + \ddot{V}_a = 12 + 0 = 12V \\ V_b &= \dot{V}_b + \ddot{V}_b = 9 - 3 = 6V \\ V_c &= \dot{V}_c + \ddot{V}_c = 9 + 7 = 16V \end{aligned} \right\} \text{Ans}$$



$i = ?$, by superposition



$$V_c = 0, \quad V_b = V_a + E_1$$

$$\frac{V_a}{R_1} + \frac{V_b}{R_2} + \frac{V_b}{R_3} = 0$$

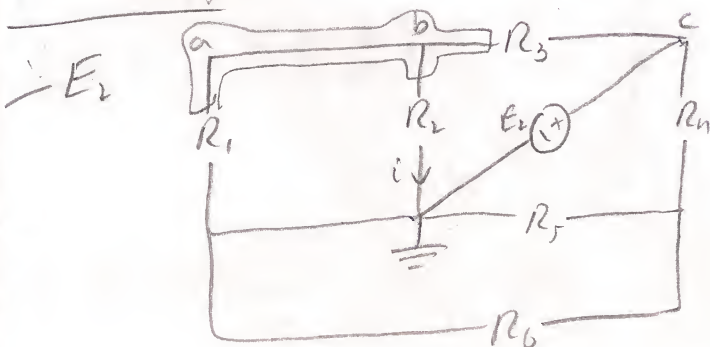
$$\frac{V_b - E_1}{R_1} + \frac{V_b}{R_2} + \frac{V_b}{R_3} = 0$$

$$V_b \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{30} \right) = 1$$

$$V_b \left(\frac{6+3+2}{60} \right) = 1$$

$$V_b = \frac{60}{11} \Rightarrow V_b = 5.45V$$

$$V_b = 5.45V$$



$$V_c = 20V, \quad V_a = V_b$$

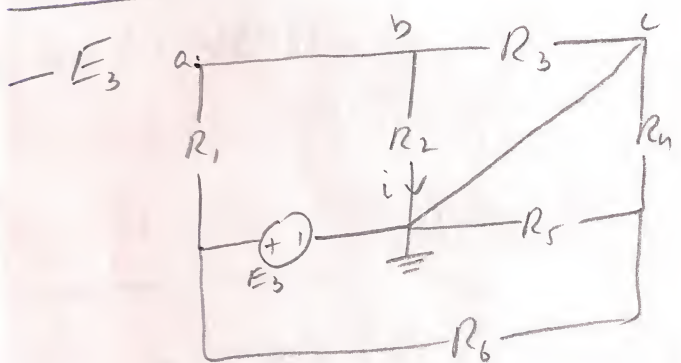
$$\frac{V_b}{R_1} + \frac{V_b}{R_2} + \frac{V_b - 20}{R_3} = 0$$

$$V_b \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{30} \right) = \frac{2}{3}$$

$$V_b \left(\frac{11}{60} \right) = \frac{2}{3}$$

$$V_b = \frac{40}{11}$$

$$V_b = 3.63V$$



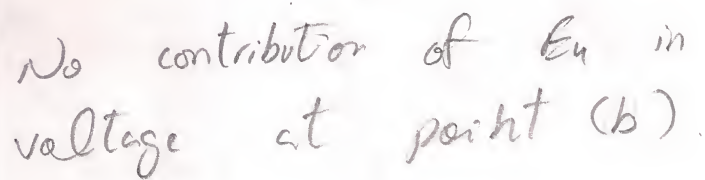
$$V_c = 0, \quad V_a = V_b$$

$$\frac{V_b - 30}{10} + \frac{V_b}{20} + \frac{V_b}{30} = 0$$

$$V_b \left(\frac{11}{60} \right) = 3$$

$$V_b = \frac{180}{11}$$

$$V_b = 16.36V$$



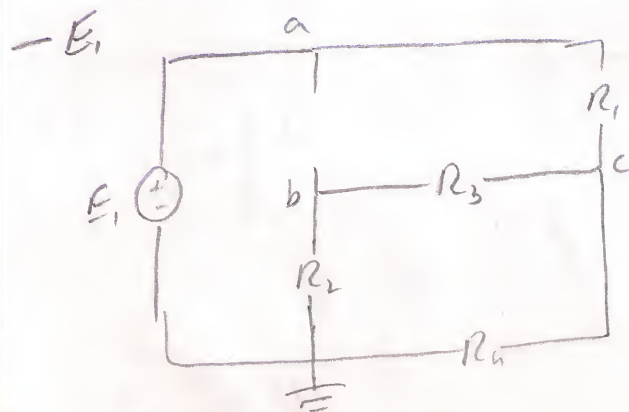
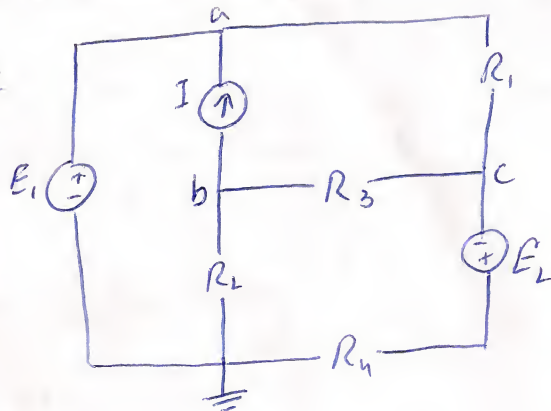
Hence

$$V_b = 5.45 + 3.63 + 16.36$$

$$V_b = 25.4436 \text{ V}$$

$$i = \frac{25.4436}{20}$$

2-16



$$V_a = E_1 = 60 \text{ V}$$

$$b) \frac{V'_b - V'_c}{R_3} + \frac{V'_b}{R_2} = 0$$

$$\check{V}_b \left(\frac{1}{30} + \frac{1}{2} \right) = \frac{\check{V}_c}{30}$$

$$V_b = \frac{1 + 15}{30} = \frac{V_c}{30}$$

$$\dot{V}_b = \frac{\dot{V}_c}{16} \quad \text{--- (1)}$$

$$c) \frac{\dot{V}_c - 60}{R_1} + \frac{\dot{V}_c - \dot{V}_b}{R_3} + \frac{\dot{V}_c}{R_n} = 0$$

$$V_c \left(\frac{1}{1} + \frac{1}{30} - \frac{1}{480} + \frac{1}{10} \right) - 60 = 0$$

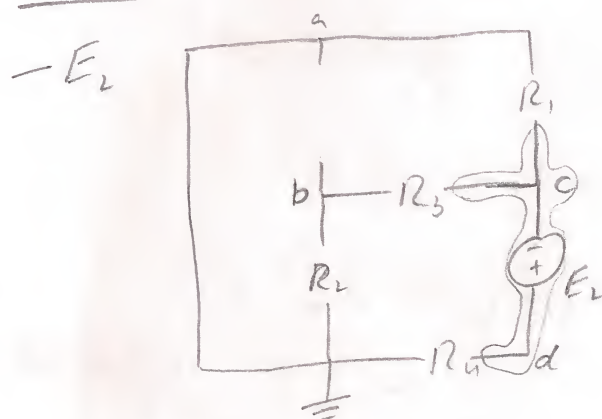
$$\checkmark_c \left(\frac{480 + 16 - 1 + 48}{480} \right) = 60$$

$$\sqrt{c}(1.131) \approx 60$$

$$V_c = 53.0386 \text{ V}$$

$$V_b = 3.3149 \text{ V}$$

$$V_a \approx 60 \text{ V}$$



$$\ddot{V}_a = 0, \quad \ddot{V}_d = \ddot{V}_c + 30$$

$$b) \frac{V_b''}{2} + \frac{V_b'' - V_c''}{R_3} = 0$$

$$\checkmark_b \left(\frac{16}{30} \right) = \frac{\checkmark_c}{30}$$

$$V_b'' = \frac{V_c'}{1.6} = \text{---} \quad (1)$$

$$\frac{V_c''}{1} + \frac{V_c'' - V_b''}{30} + \frac{V_c'' + 30}{R_u} = 0$$

$$V_c'' \left(1 + \frac{1}{30} - \frac{1}{480} + \frac{1}{10} \right) = -3$$

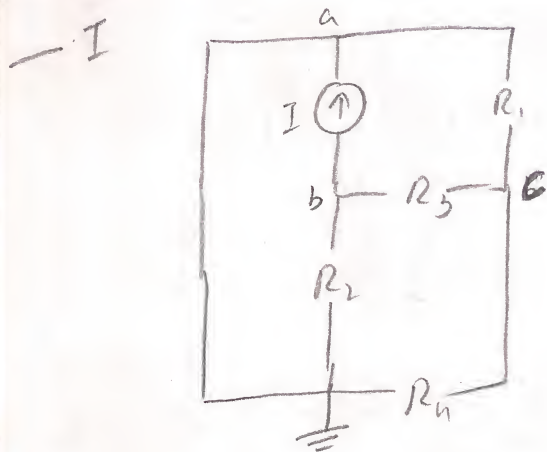
$$V_c'' \left(\frac{543}{480} \right) = -3$$

$$V_c'' = \frac{-3}{1.131}$$

$$V_c'' = -2.6525 \text{ V}$$

$$V_b'' = -0.1657 \text{ V}$$

$$V_a'' = 0 \text{ V}$$



$$V_a''' = 0$$

$$b) \cdot 7 + \frac{V_b'' - V_c''}{30} + \frac{V_b''}{2} = 0$$

$$V_b''' \left(\frac{1}{30} + \frac{1}{2} \right) = \frac{V_c''}{30} - 7$$

$$V_b''' \left(\frac{16}{30} \right) = \frac{V_c'' - 216}{30}$$

$$V_b''' = \frac{V_c'' - 216}{16}$$

$$c) \frac{V_c''}{1} + \frac{V_c'' - V_b''}{30} + \frac{V_c''}{10} = 0$$

$$V_c''' \left(1 + \frac{1}{30} + \frac{1}{10} \right) - \frac{V_c''}{16 \cdot 30} + \frac{216}{480} = 0$$

$$V_c''' \left(1 + \frac{1}{30} - \frac{1}{480} + \frac{1}{10} \right) = -\frac{216}{480}$$

$$V_c''' (1.131) = -0.45$$

$$V_c''' = -0.3978$$

$$V_b''' = -13.52 \text{ V}$$

$$V_a''' = 0$$

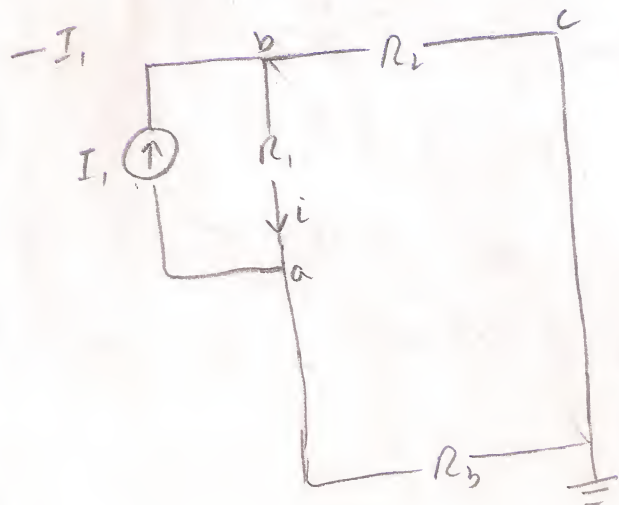
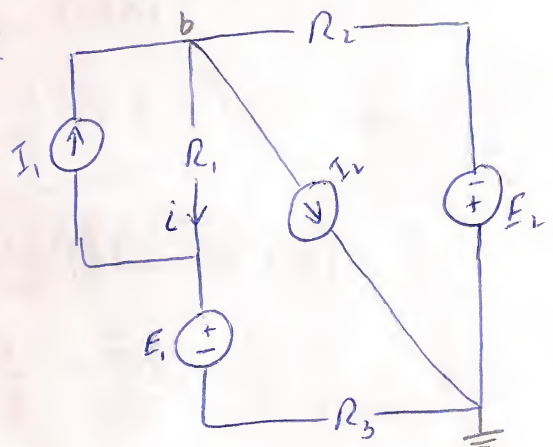
Hence

$$V_a = 60 + 0 + 0 = 60 \text{ V}$$

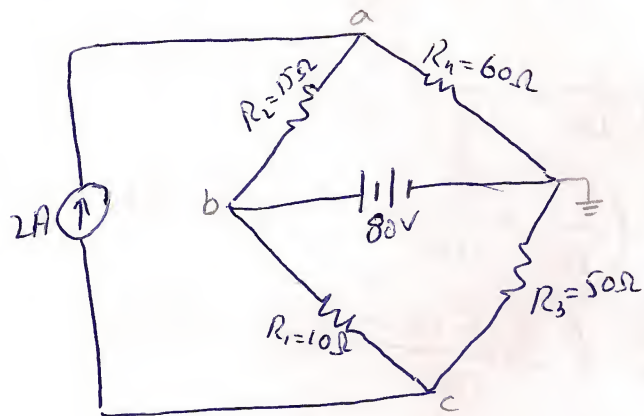
$$V_b = 3.3149 - 0.1657 - 13.52 = -10 \text{ V}$$

$$V_c = 53.0386 - 2.6525 - 0.3978 = 50 \text{ V}$$

2.17



2.18 $P(R_2) = ?$



$V_b = 80V$

a) $\frac{V_a - 80}{15} - 2 + \frac{V_a}{60} = 0$

$V_a \left(\frac{1}{15} + \frac{1}{60} \right) - \frac{80}{15} - 2 = 0$

$V_a \left(\frac{4+1}{60} \right) = \frac{16+6}{3}$

$V_a = \frac{22 \times 80}{5}$

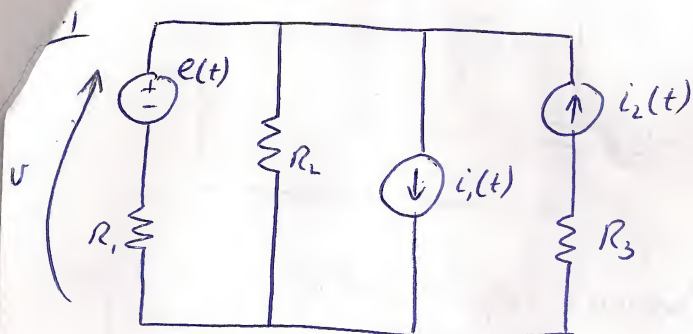
$V_a = 88V$

$P(R_2) = R_2 \left(\frac{V_a - V_b}{R_2} \right)^2$

$= 15 \left(\frac{8}{15} \right)^2 = \frac{64}{15}$

$\boxed{P(R_2) = 4.26 W} \quad \underline{\text{Ans}}$

CH#3



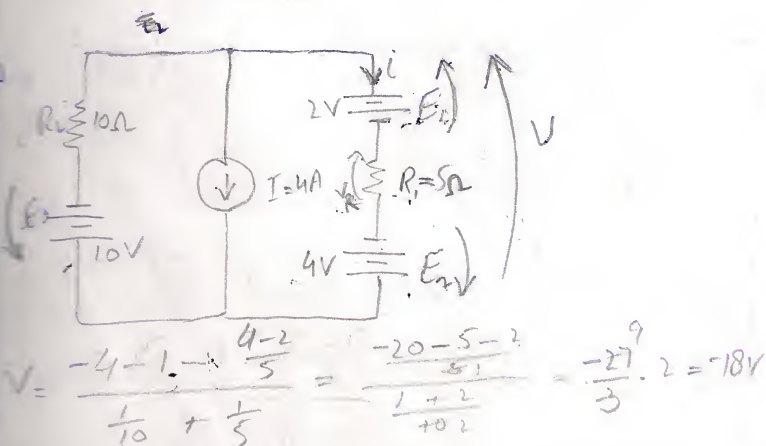
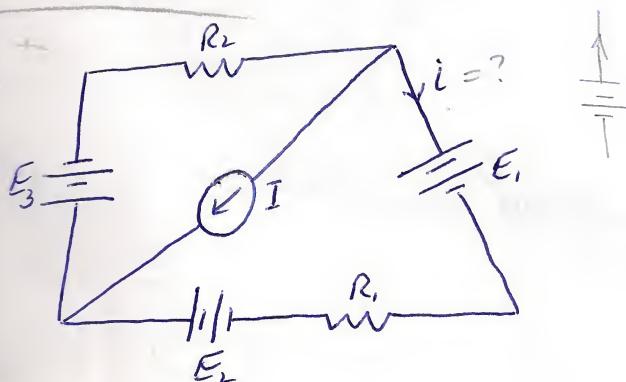
$$V = \frac{\frac{e(t)}{R_1} - i_1(t) + i_2(t)}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$= \frac{\frac{20}{1/2} - 7 + 2}{\frac{1}{1/2} + \frac{1}{1/3}}$$

$$= \frac{20 - 7 + 2}{2 + 3} = \frac{15}{5}$$

$$V = 3V$$

3.2



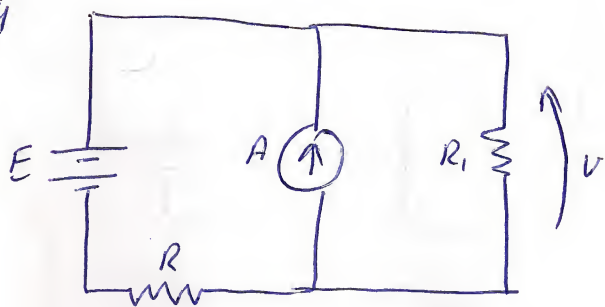
$$V = \frac{-4 - 1 \cdot \frac{4-2}{5}}{\frac{1}{10} + \frac{1}{5}} = \frac{-20 - 5 - 2}{\frac{1+2}{10}} = \frac{-27}{3} \cdot 2 = -18V$$

$$-18 - 2 - V_R + 4 = 0 \Rightarrow V_R = -16$$

$$i = \frac{V_R}{R_1} = \frac{-16}{5} \Rightarrow i = -3.2A$$

$$i = i_L - I \Rightarrow i = -3.2A$$

3.4

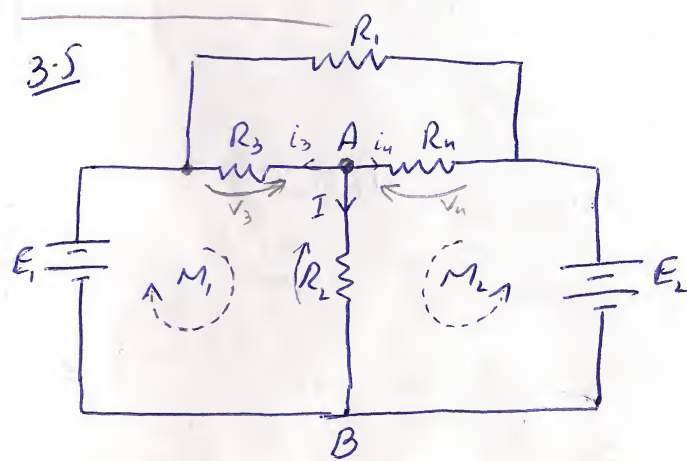


$$V = \frac{\frac{E}{R} + A}{\frac{1}{R} + \frac{1}{R_1}}$$

$$V = \frac{E + RA}{R(\frac{R_1 + R}{RR_1})}$$

$$V = \frac{E}{\frac{R_1}{R}} \Rightarrow V = E$$

3.5



$$\text{KVL } M_1: E_1 + i_3 R_3 - I R_2 = 0 \quad \text{--- ①}$$

$$\text{KVL } M_2: E_2 + i_4 R_n - I R_2 = 0 \quad \text{--- ②}$$

By ① - ②

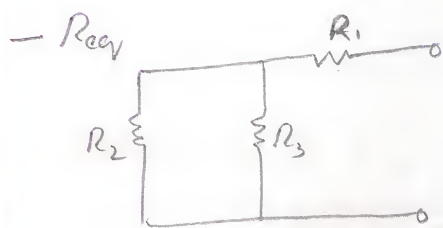
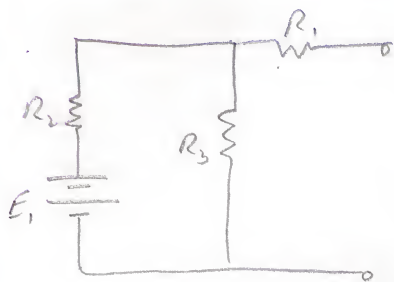
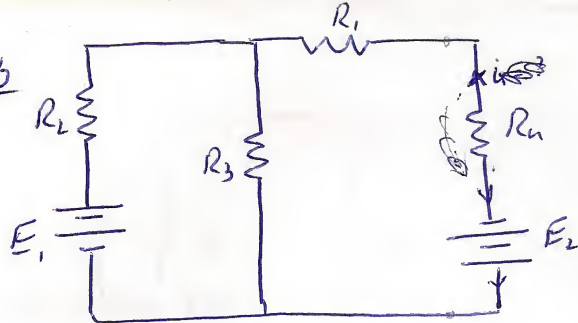
$$E_1 - E_2 + i_3 R_3 - i_4 R_n = 0$$

$$4 - 8 + i_3 2 - i_4 4 = 0$$

$$i_3 - 2i_4 = 2$$

KCL

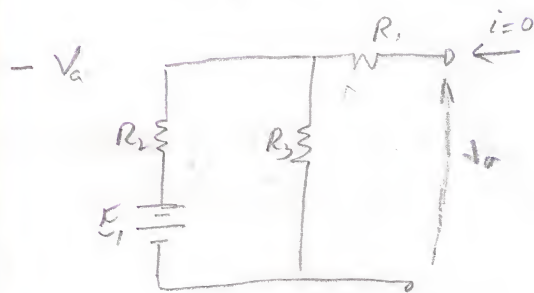
3.6



$$R_{eq} = R_2 \parallel R_3 + R_1$$

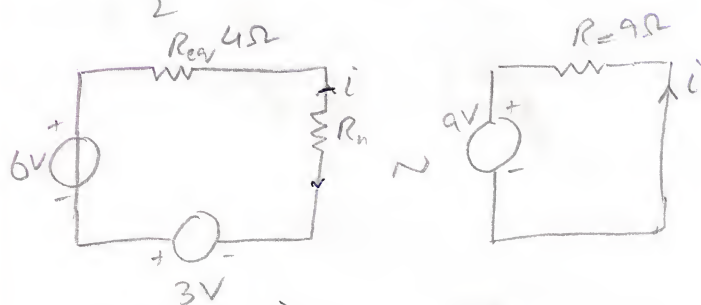
$$= \frac{4 \cdot 4}{8} + 2$$

$$= 2 + 2 = 4 \Omega$$



$$V_a = \frac{E_1}{\frac{1}{R_2} + \frac{1}{R_3}} \Rightarrow V_a = \frac{12}{\frac{1}{4} + \frac{1}{4}}$$

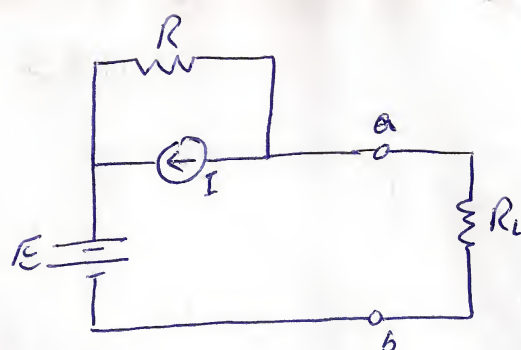
$$V_a = \frac{3}{\frac{1}{2}} \Rightarrow V_a = 6V$$



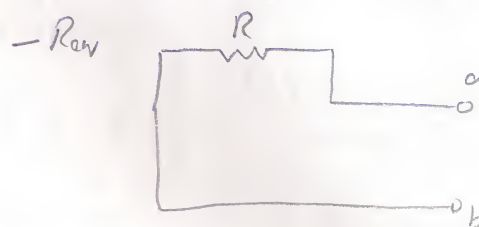
$$i = \frac{V_{oc}}{R_{eq} + R_L}$$

$$i = \frac{9}{9} \Rightarrow i = 1A$$

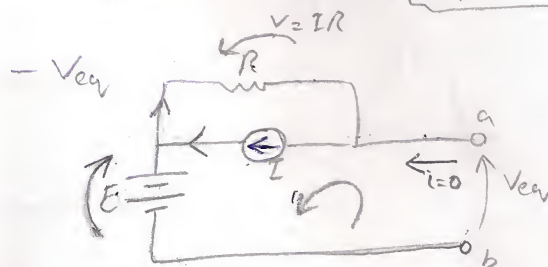
3.7



i) Thevenin's equivalent = ?



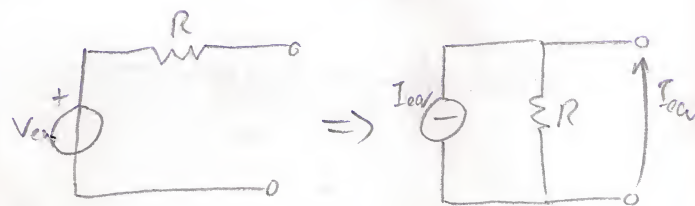
$$R_{eq} = R \Rightarrow R_{eq} = 1 \Omega$$



$$V_{eq} = E - IR$$

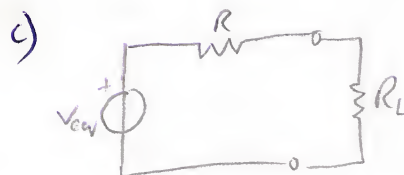
$$= 12 - 4 \Rightarrow V_{eq} = 8V$$

ii) Norton equivalent = ?



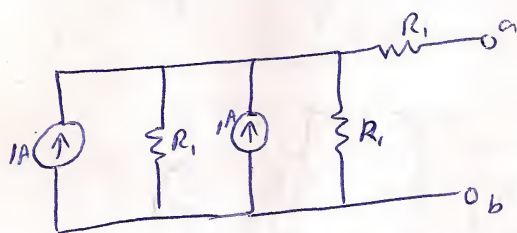
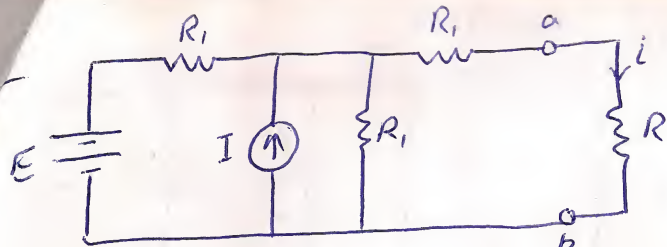
$$I_{eq} = \frac{V_{eq}}{R}$$

$$= \frac{8}{1} \Rightarrow I_{eq} = 8A$$



$$P = R_L \left(\frac{V_{eq}}{R + R_L} \right)^2 = \left(\frac{8}{1 + R_L} \right)^2 R_L$$

$$P = 64 \frac{R_L}{(1 + R_L)^2} A^2$$



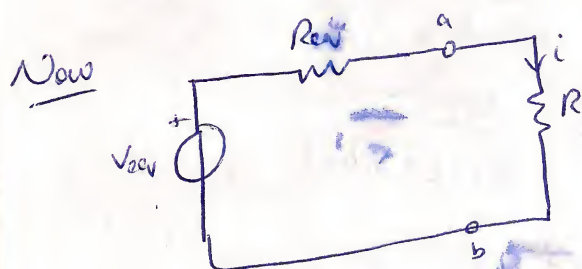
$$-R_{eq} \cdot \frac{10 \cdot 10}{20} + 10$$

$$R_{eq} = 15 \Omega$$

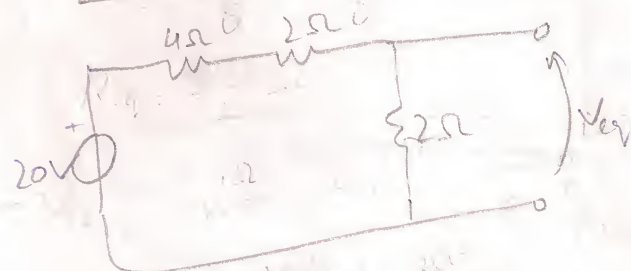
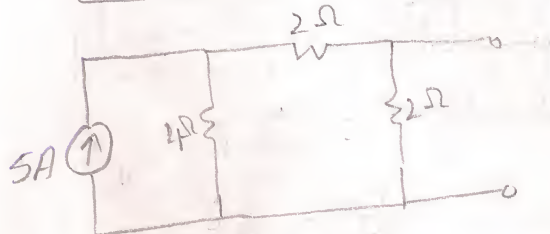
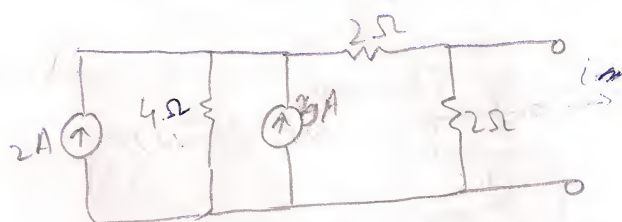
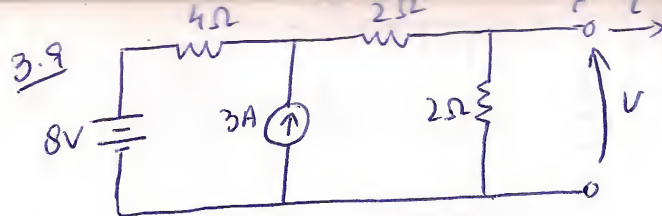


$$V_{eq} = \frac{10}{20} \times 20$$

$$V_{eq} = 10V$$



$$i = \frac{10}{15 + R} A$$



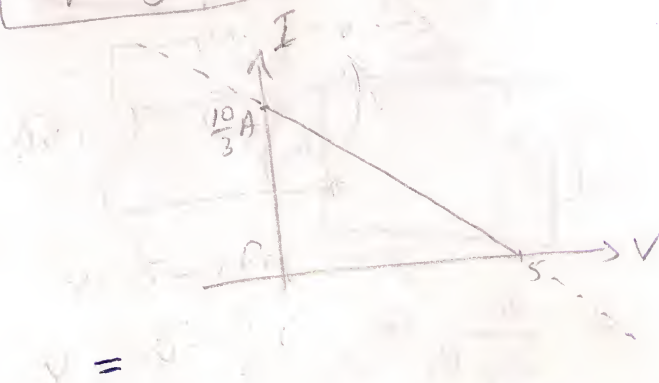
$$R_{eq} = \frac{6 \cdot 2}{8} = \frac{12}{8} = \frac{3}{2} \Omega$$

$$V_{eq} = \frac{2}{8} \times 20$$

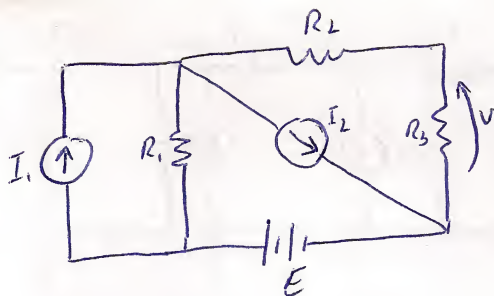
$$V_{eq} = 5V$$

$$I_{eq} = \frac{V_{eq}}{R_{eq}} = \frac{5}{3/2} = \frac{10}{3} A$$

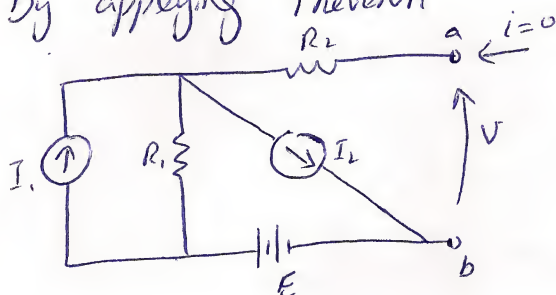
$$I_{eq} = \frac{10}{3} A$$



3.10

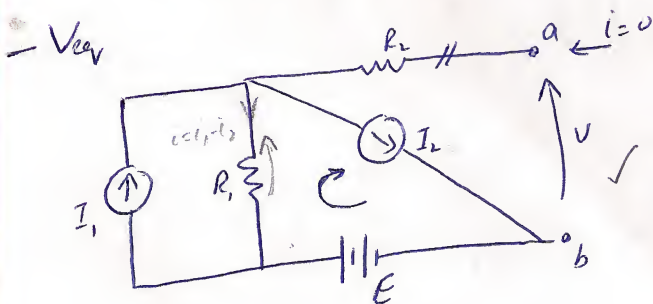


By applying Thevenin

 R_{eq} 

$$R_{eq} = R_1 + R_2$$

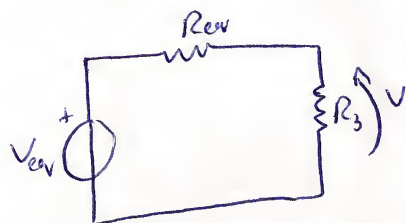
$$= 1 + 2 \Rightarrow R_{eq} = 3\Omega$$



$$V_{eq} = E + R_1(I_1 - I_2)$$

$$= 6 + 1(4 - 5)$$

$$= 6 - 1 \Rightarrow V_{eq} = 5V$$



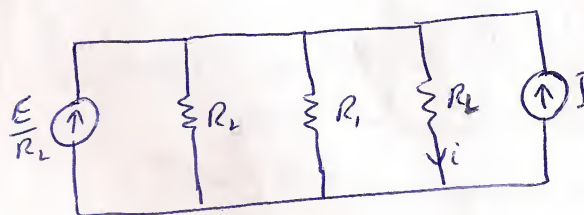
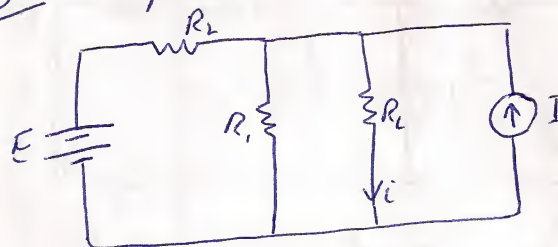
$$V = \frac{R_3}{R_{eq} + R_3} V_{eq}$$

$$= \frac{3}{3+3} \times 5$$

$$V = \frac{15}{6}$$

$$\Rightarrow V = 2.5V$$

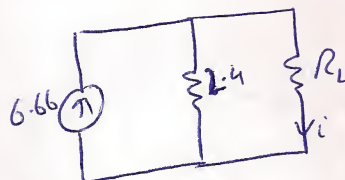
3.11

compute i 

$$I_{eq} = \frac{E}{R_1} + I \Rightarrow I_{eq} = \frac{10}{6} + 5 = \frac{40}{6}$$

$$I_{eq} = 6.66A$$

$$R_1 || R_2 = \frac{24}{10} = 2.4\Omega$$



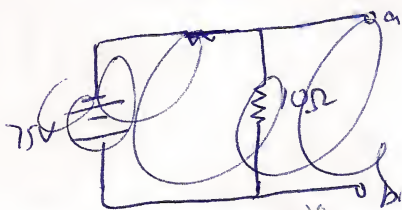
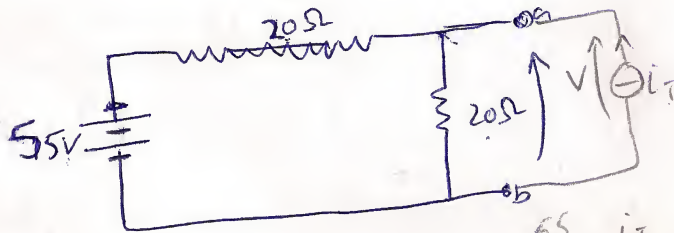
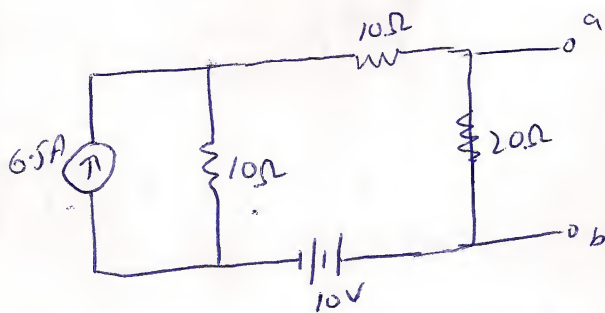
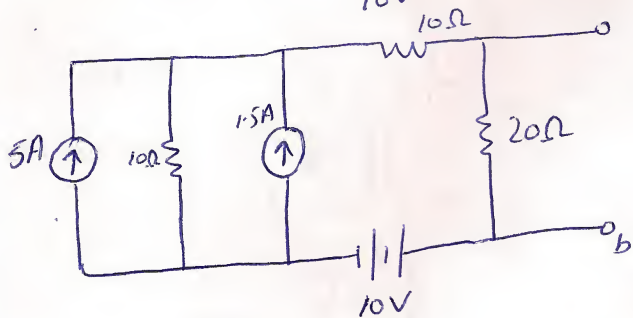
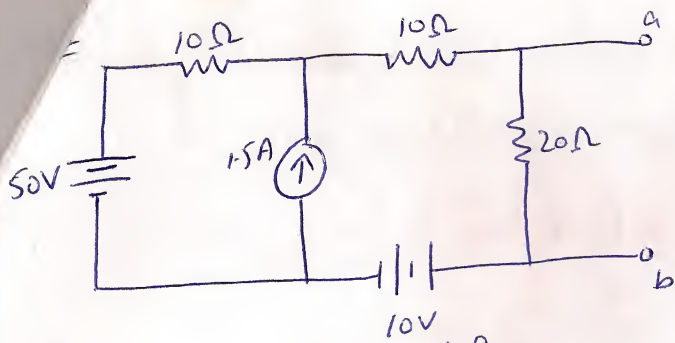
$$i = \frac{2.4}{2.4 + R_L} \times 6.66$$

$$i = \frac{\frac{24}{10}}{\frac{24}{10} + R_L} \times \frac{40}{6}$$

$$= \frac{244}{24 + 10R_L} \times \frac{40}{6}$$

$$= \frac{16080}{2(12 + 5R_L)}$$

$$i = \frac{80}{12 + 5R_L} A$$



$$V = \frac{55}{20} + \frac{i_T}{10}$$

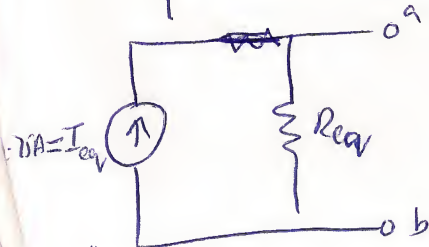
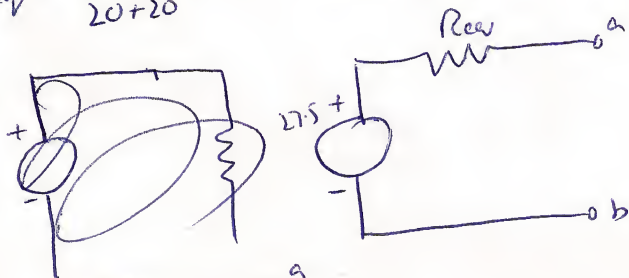
$$V = \frac{55}{2} + 10i_T$$

$$V_{eq} = 27.5V$$

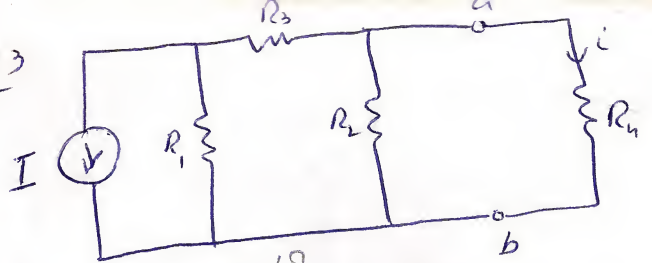
$$R_{eq} = 10\Omega$$

$$R_{eq} = 10\Omega$$

$$V_{eq} = \frac{20}{20+20} \times 55 \Rightarrow V_{eq} = 27.5V$$



3.13

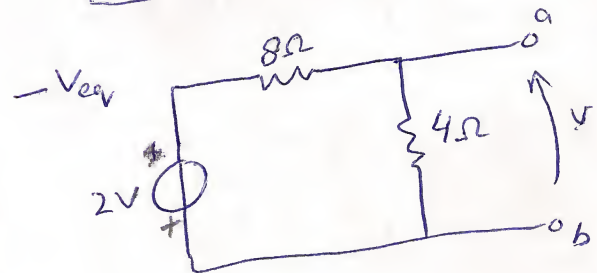


$-R_{eq}$

$$R_1 + R_3 \parallel R_2$$

$$R_{eq} = \frac{8 \cdot 8}{8+8} = \frac{32}{16} = 2\Omega$$

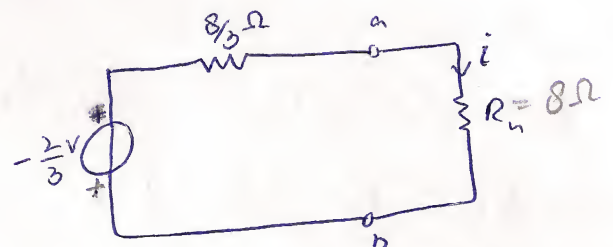
$$R_{eq} = \frac{8}{3}\Omega$$



$$V_{eq} = -\frac{4}{12} \times 2$$

$$= -\frac{8}{12}$$

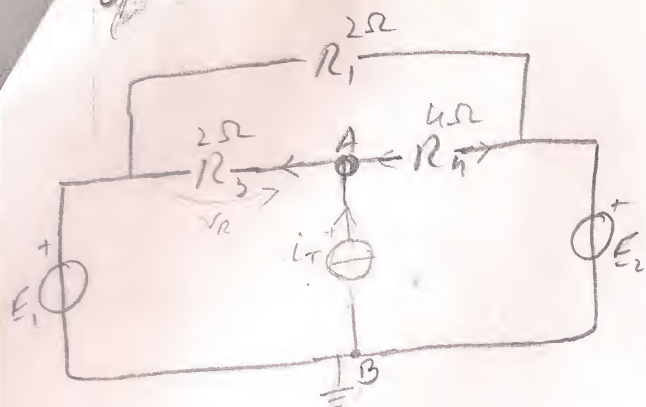
$$V_{eq} = -\frac{2}{3}V$$



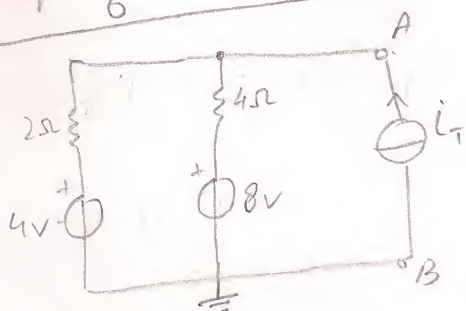
$$i = \frac{-2/3}{8/3 + 8} = \frac{-2}{32}$$

$$i = \frac{-1}{16}A$$

by Thevenin



$$R_{eq} = \frac{8}{6} = 1.33 \Omega$$



KCL

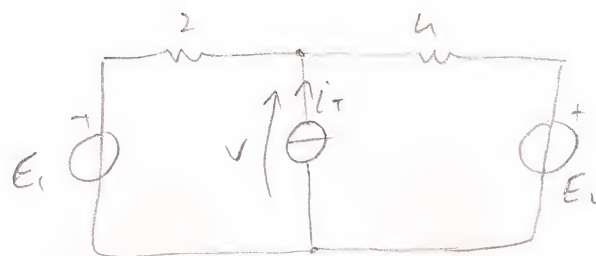
$$\frac{4}{2} + \frac{8}{2} - i_T = 0$$

$$i_T = 4A$$

$$V_{eq} = i_T R_{eq}$$

$$= 4(1.33)$$

$$V_{eq} = 5.3V$$



$$\frac{V-E_1}{2} + \frac{V-E_2}{4} - i_T = 0$$

$$V\left(\frac{1}{2} + \frac{1}{4}\right) - 2 - 2 - i_T = 0$$

$$V\left(\frac{3}{4}\right) = 4 + i_T$$

$$V = 5.33 + 1.33 i_T$$

$$V_{eq} = 5.33V$$

$$R_{eq} = 1.33 \Omega$$

OR

$$V = \frac{\frac{4}{2} + \frac{8}{2} + i_T}{\frac{1}{2} + \frac{1}{4}} = \frac{4 + i_T}{\frac{3}{4}}$$

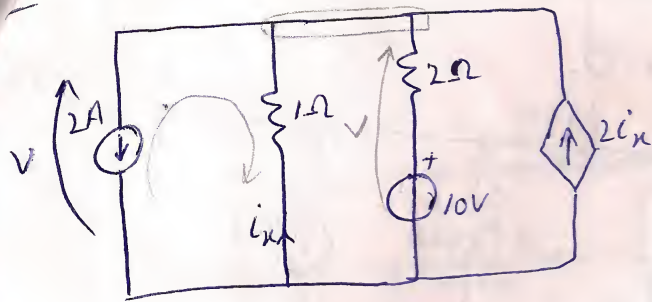
$$V = \frac{16}{3} + \frac{4}{3} i_T$$

$$R_{eq} = \frac{4}{3} \Omega$$

$$V_{eq} = \frac{16}{3} V$$

CIRCUITS WITH DEPENDENT SOURCES

4.1 $V = ?$



KCL $2 - i_x + \frac{V - 10}{2} - 2i_x = 0$

$$\frac{V}{2} - 5 + 2 - 3i_x = 0$$

$$\frac{V}{2} = 3(i_x + 1)$$

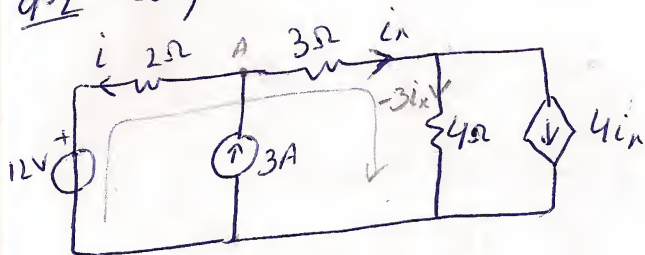
KVL $V + i_x = 0 \Rightarrow i_x = -V$

$$\frac{V}{2} = -3V + 3$$

$$\frac{V}{2} + 3V = 3 \Rightarrow 7V = 6$$

$$V = \frac{6}{7} \Rightarrow \boxed{V = 0.8571V}$$

4.2 compute i_i



KCL @ A
 $i + i_x = 3 \Rightarrow i_x = 3 - i$

KVL $12 + 2i - 3i_x - 4(-3i_x) = 0$

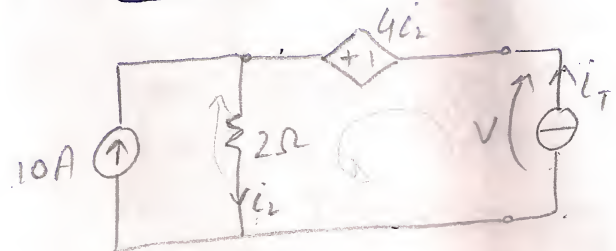
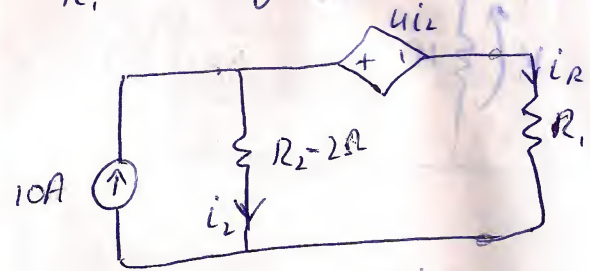
$$12 + 2i - 3i_x + 12i_x = 0$$

$$12 + 2i + 9i_x = 0$$

$$12 + 2i + 27 - 9i = 0$$

$$7i = 39 \Rightarrow \boxed{i = 5.57A}$$

4.3 Evaluate i_R as a function of R_1 using Thevenin's theorem.

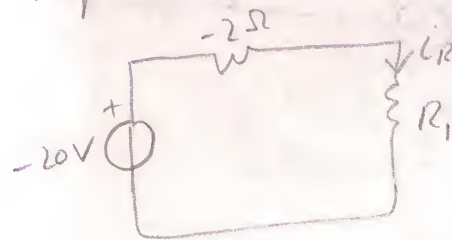


KCL $-10 + i_2 - i_T = 0$
 $i_2 = i_T + 10$

KVL $V + 4i_2 - 2i_2 = 0$
 $V + 2(i_T + 10) = 0$
 $V = -20 - 2i_T$

$$V_{oc} = -20V$$

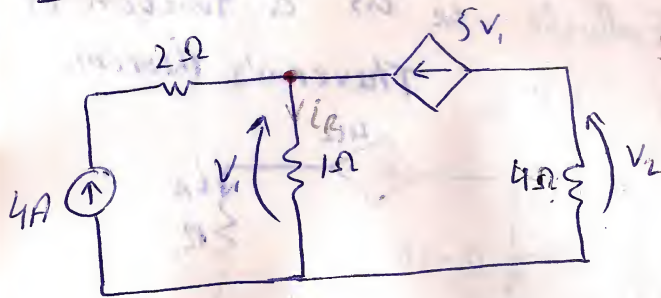
$$R_{eq} = -2\Omega$$



$$i_R = \frac{-20}{-2 + R_1}$$

$$\boxed{i_R = \frac{20}{2 - R_1} A}$$

4.4 compute V_2



KCL $-4 + \frac{V_1}{1} - 5V_1 = 0 \Rightarrow -4V_1 = 4$

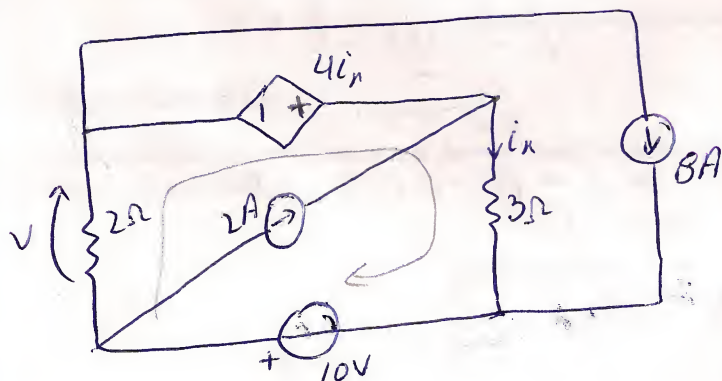
$V_1 = -1V$

KVL $V_2 = -5V_1(4)$

$V_2 = -5(-1)(4)$

$V_2 = 20V$

4.5 Compute V



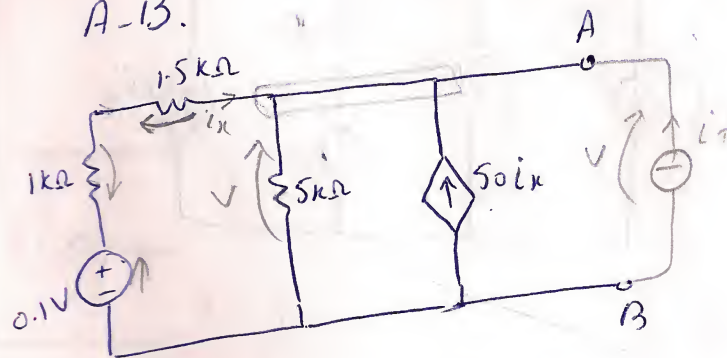
KCL $8 + \frac{V}{2} + i_x - 2 = 0$
 $i_x = -6 - \frac{V}{2}$

KVL $V + 4i_x - 3i_x + 10 = 0$
 $V - 6 - \frac{V}{2} + 10 = 0$

$\frac{V}{2} = -4$

$V = -8V$

4.6 Build thevenin equivalent for the circuit of Terminal A-B.



KCL $-i_T - 50i_x + \frac{V}{5 \times 10^3} - i_x = 0$

$51i_x = \frac{V}{5 \times 10^3} - i_T$

$i_x = \left(\frac{V - 5i_T}{5k} \right) \frac{1}{51}$

KVL $1 - 5i_x + i_x - 0.1 + V = 0$

$\frac{2 \cdot 5k(V - 5i_T)}{51} + V = 0.1$

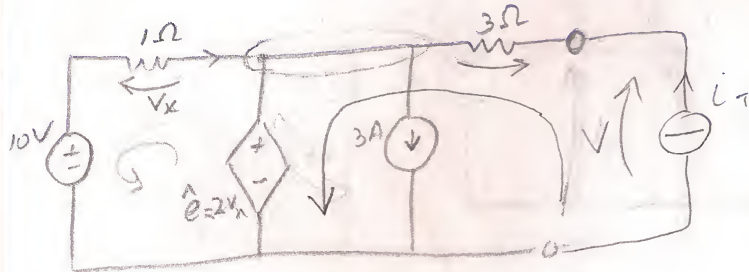
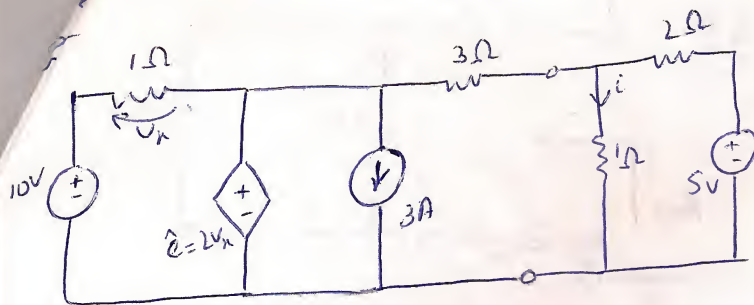
$0.0098036V - 49.019i_T + V = 0.1$

$1.0098036V = 0.1 + 49.019i_T$

$V = 0.0099 + 48.54i_T$

$V_{eq} = 0.0099V$

$R_{eq} = 48.54\Omega$



KVL

$$V_x - 10 + 2V_x = 0$$

$$3V_x = 10 \Rightarrow V_x = \frac{10}{3}$$

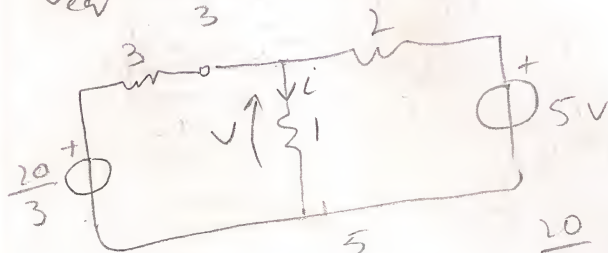
$(-2V_x)$

$$V - 3i_T - \frac{20}{3} = 0$$

$$V = 3i_T + \frac{20}{3}$$

$R_{eq} = 3\Omega$

$V_{eq} = \frac{20}{3}V$

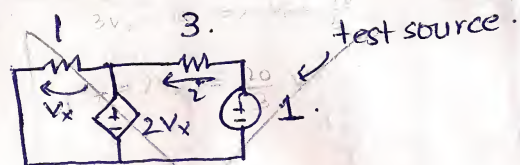


$$V = \frac{\frac{20}{3}/3 + \frac{5}{2}}{\frac{1}{3} + 1 + \frac{1}{2}} = \frac{\frac{20}{9} + \frac{5}{2}}{\frac{2+6+3}{6}}$$

$$V = \frac{\frac{40+45}{18}}{\frac{11}{6}} = \frac{85}{33}V$$

$$i = \frac{V}{1} \Rightarrow i = 2.57A$$

for Req

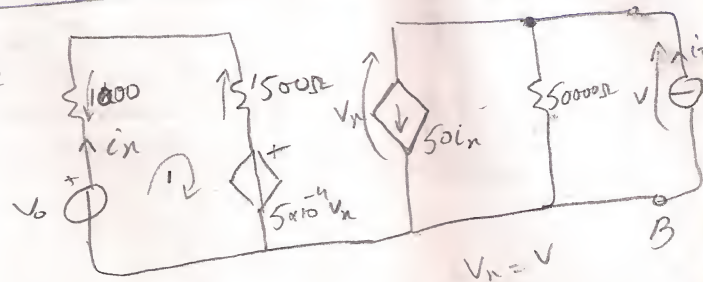


$$\frac{2V_x - 0}{1} + \frac{2V_x - 1}{3} = 0$$

$$V_x = \frac{1}{8}$$

$$i = \frac{1 - 2V_x}{3} = \frac{1 - \frac{1}{4}}{3} = \frac{1}{3}A$$

4.8



KVL(1)

$$V_0 - i_n(2500) - 0.0005V_x = 0$$

$$i_n = \frac{V_0}{2500} - \frac{0.0005V_x}{2500}$$

KCL

$$50i_n + \frac{V}{50000} = i_T$$

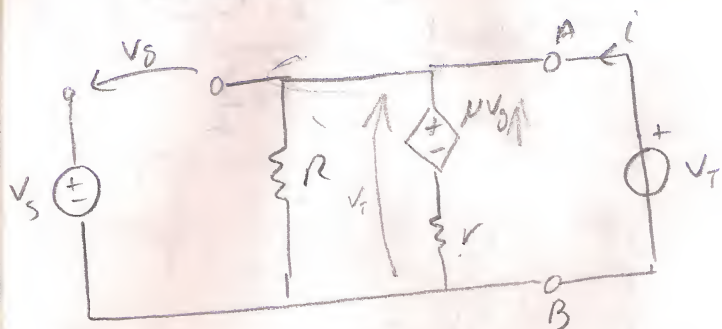
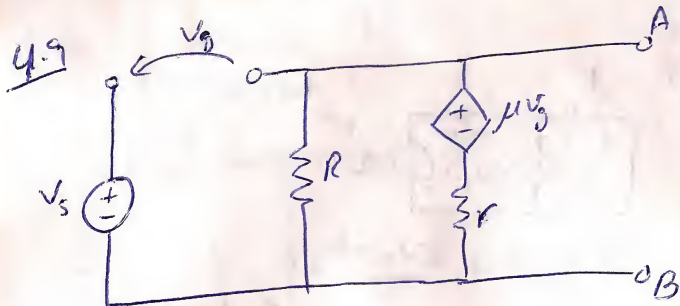
$$50 \left(\frac{V_0 - 0.0005V}{2500} \right) + \frac{V}{50000} = i_T$$

$$\frac{V_0}{50} - 1 \times 10^{-5}V + \frac{V}{50000} = i_T$$

$$\frac{-0.5 + 1}{50000} V = i_T - \frac{V_0}{50}$$

$$V = 100000i_T - \frac{V_0}{5 \times 10^{-4}}$$

$R_{eq} = 100k\Omega$



KVL

$$V_T + V_0 = V_s$$

$$V_0 = V_s - V_T$$

KCL

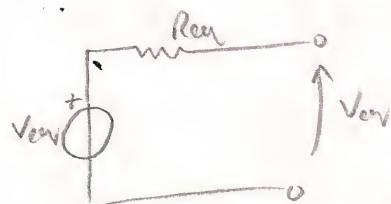
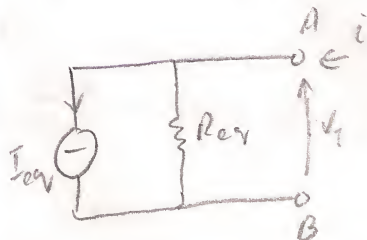
$$-i + \frac{V_T}{R} + \frac{V_T - \mu V_0}{r}$$

$$i = \frac{V_T}{R} + \frac{V_T}{r} - \frac{\mu(V_s - V_T)}{r}$$

$$i = V_T \left(\frac{1}{R} + \frac{1+\mu}{r} \right) - \frac{\mu V_s}{r}$$

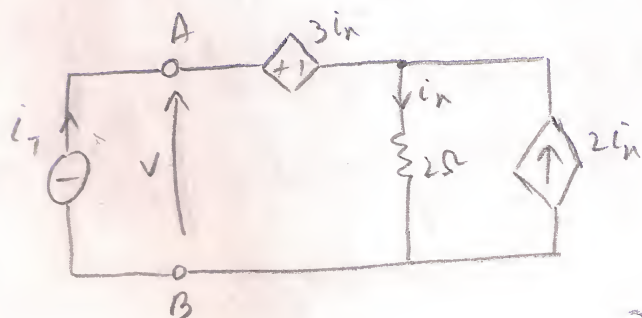
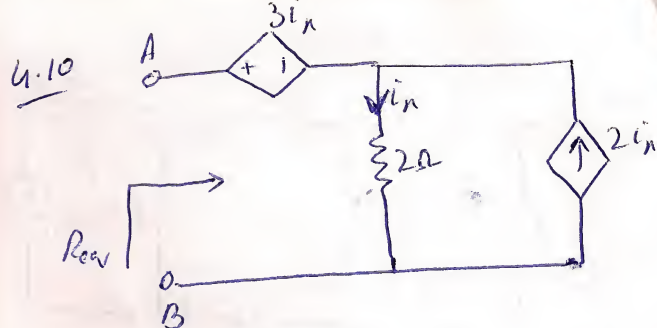
G_{eq}

$$G_{eq} = \frac{r + R(1+\mu)}{rR}, \quad I_{eq} = -\frac{\mu V_s}{r}$$



$$R_{eq} = \frac{rR}{r + R(1+\mu)} \quad V_{eq} = \frac{\mu V_s R}{r + R(1+\mu)}$$

$$V_{eq} = -I_{eq} R_{eq} = \frac{\mu V_s}{r} \cdot \frac{rR}{r + R(1+\mu)}$$



$$V_{eq} = 0 \quad (\text{No independent source})$$

KCL

$$i_T = i_n - 2i_n$$

$$i_T = -i_n \Rightarrow i_n = -i_T$$

KVL

$$V = 3i_n + 2i_n$$

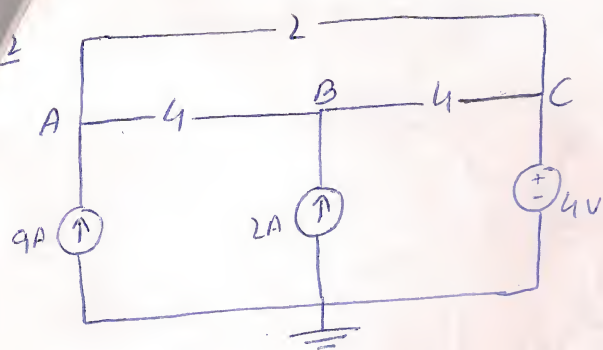
$$V = 5i_n$$

$$V = -5i_T$$

$$R_{eq} = \frac{V}{i_T} = \frac{-5i_T}{i_T}$$

$$R_{eq} = -5\Omega$$

$$V = R_{eq} + I_T V_{eq}$$



$$V_C = 4V$$

$$A) \frac{V_A - V_B}{4} - 9 + \frac{V_A - 4}{2} = 0$$

$$V_A \left(\frac{1}{4} + \frac{1}{2} \right) = 11 + \frac{V_B}{4}$$

$$V_A = \frac{44}{3} + \frac{V_B}{3}$$

$$B) \frac{V_B - 4}{4} + \frac{V_B - V_A}{4} - 2 = 0$$

$$V_B \left(\frac{1}{4} + \frac{1}{4} \right) - \frac{11}{3} - \frac{V_B}{12} - 2 = 0$$

$$V_B \left(\frac{1}{4} + \frac{1}{4} - \frac{1}{12} \right) = \frac{20}{3}$$

$$V_B \left(\frac{3 + 3 - 1}{12} \right) = \frac{20}{3}$$

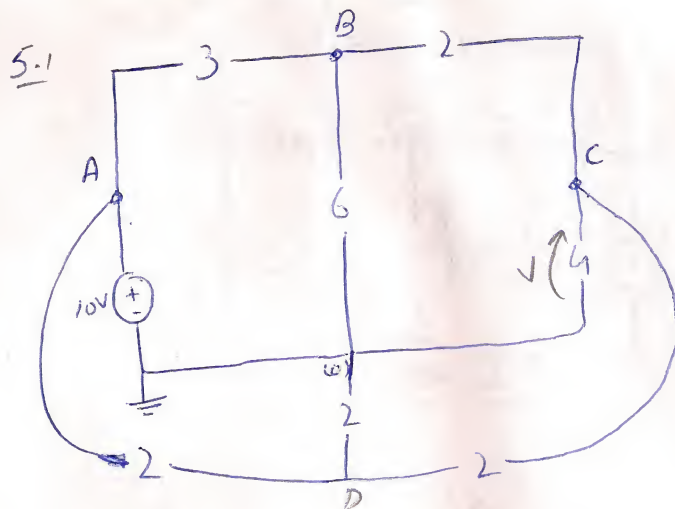
$$V_B = \frac{80}{5}$$

$$\boxed{V_B = 16V} \quad \text{B}$$

$$V_A = \frac{44}{3} + \frac{16}{3}$$

$$V_A = \frac{60}{3}$$

$$\boxed{V_A = 20V} \quad \text{A}$$



$$V_A = 10V$$

$$B) \frac{V_B - 10}{3} + \frac{V_B}{6} + \frac{V_B - V_C}{2} = 0$$

$$V_B \left(\frac{1}{3} + \frac{1}{6} + \frac{1}{2} \right) = \frac{10}{3} + \frac{V_C}{2}$$

$$V_B \left(\frac{2 + 1 + 3}{6} \right) = \frac{10}{3} + \frac{V_C}{2}$$

$$V_B = \frac{10}{3} + \frac{V_C}{2} \quad \checkmark$$

$$C) \frac{V_C}{4} + \frac{V_C - V_D}{2} + \frac{V_C - V_B}{2} = 0$$

$$V_C \left(\frac{1}{4} + \frac{1}{2} + \frac{1}{2} \right) - \frac{10}{6} - \frac{V_C}{4} = 0$$

$$V_C = \frac{5}{3} + \frac{V_D}{2} \quad \checkmark$$

$$D) \frac{V_D - 10}{2} + \frac{V_D}{2} + \frac{V_D - V_C}{2} = 0$$

$$V_D \left(\frac{1}{2} + \frac{1}{2} + \frac{1}{2} \right) - \frac{5}{6} - \frac{V_D}{4} = 5$$

$$V_D \left(\frac{2 + 2 + 2 - 1}{4} \right) = 5 + \frac{5}{6}$$

$$V_D = \left(5 + \frac{5}{6} \right) \frac{4}{5}$$

$$V_D = 4 + \frac{2}{3}$$

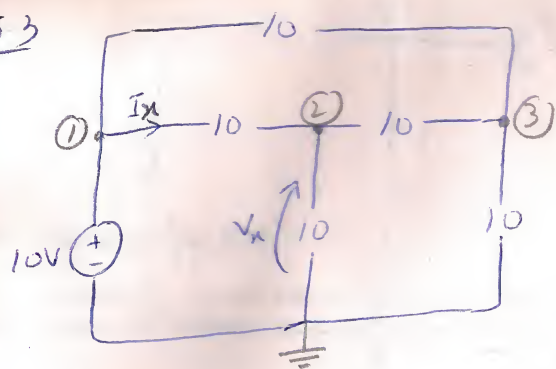
$$V_D = \frac{14}{3} \quad \checkmark$$

$$V_C = \frac{5}{3} + \frac{14}{6} = \frac{10 + 14}{6} = \frac{24}{6}$$

$$V_C = 4V$$

$$V = V_C - 0 \Rightarrow \boxed{V = 4V}$$

5.3



$$V_1 = 10V$$

$$2) \frac{V_2 - 10}{10} + \frac{V_2}{10} + \frac{V_2 - V_3}{10} = 0$$

$$V_2 \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10} \right) - \frac{V_3}{10} = 1$$

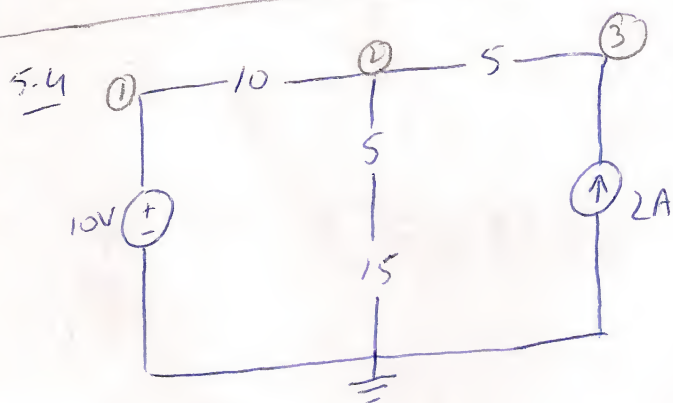
$$\frac{3}{10} V_2 - \frac{V_3}{10} = 1$$

$$3) \frac{V_3 - 10}{10} + \frac{V_3 - V_2}{10} + \frac{V_3}{10} = 0$$

$$V_3 \left(\frac{1}{10} + \frac{1}{10} + \frac{1}{10} \right) - \frac{V_2}{10} = 1$$

$$-\frac{V_2}{10} + \frac{3}{10} V_3 = 1$$

$$\begin{pmatrix} 3/10 & -1/10 \\ -1/10 & 3/10 \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$$



$$V_1 = 10V$$

$$2) \frac{V_2 - 10}{10} + \frac{V_2}{20} + \frac{V_2 - V_3}{5} = 0$$

$$V_2 \left(\frac{1}{10} + \frac{1}{20} + \frac{1}{5} \right) - \frac{V_3}{5} = 1$$

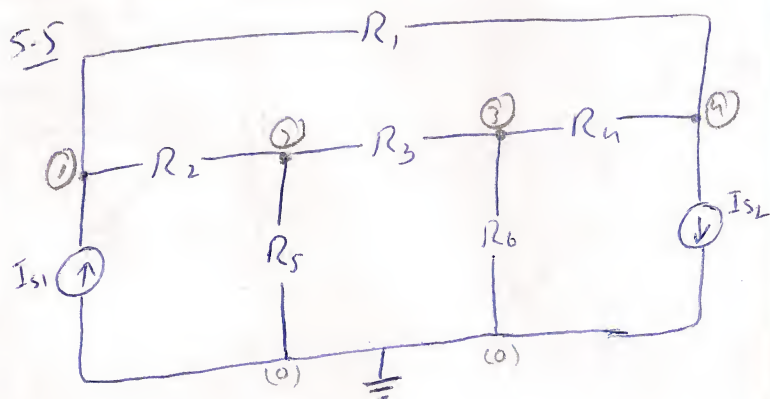
$$V_2 \frac{7}{20} - \frac{V_3}{5} = 1$$

$$3) -2 + \frac{V_3 - V_2}{5} = 0$$

$$-\frac{V_2}{5} + \frac{V_3}{5} = 2$$

$$\begin{pmatrix} 7/20 & -1/5 \\ -1/5 & 1/5 \end{pmatrix} \begin{pmatrix} V_2 \\ V_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \end{pmatrix} \quad \underline{\text{Ans}}$$

5.5



$$1) \frac{V_1 - V_4}{1000} + \frac{V_1 - V_2}{1000} - 100 = 0$$

$$\frac{1}{1000} (2V_1 - V_2 - V_4) = 100$$

$$2) \frac{V_2}{1000} + \frac{V_2 - V_1}{1000} + \frac{V_2 - V_3}{1000} = 0$$

$$\frac{1}{1000} (-V_1 + 3V_2 - V_3) = 0$$

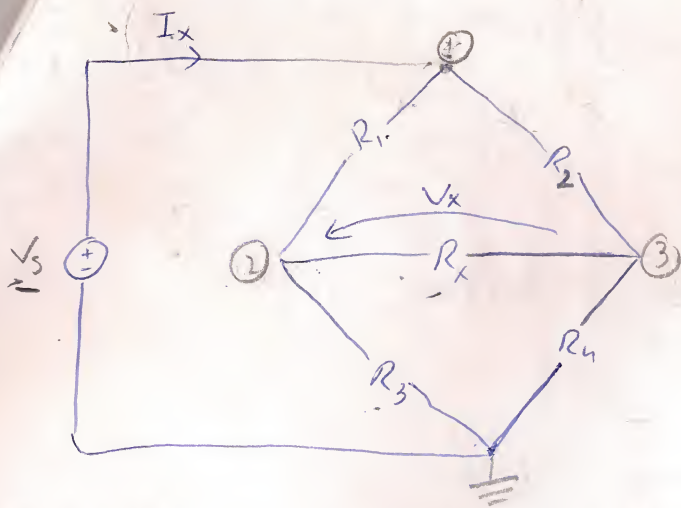
$$3) \frac{V_3 - V_2}{1000} + \frac{V_3 - V_4}{1000} + \frac{V_3}{1000} = 0$$

$$\frac{1}{1000} (-V_2 + 3V_3 - V_4) = 0$$

$$4) \frac{V_4 - V_1}{1} + \frac{V_4 - V_3}{1} + 100 = 0$$

$$\frac{1}{1000} (-V_1 - V_3 + 2V_4) = -100$$

$$\frac{1}{1000} \begin{pmatrix} 2 & -1 & 0 & -1 \\ -1 & 3 & -1 & 0 \\ 0 & -1 & 3 & -1 \\ -1 & 0 & -1 & 2 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{pmatrix} = \begin{pmatrix} 100 \\ 0 \\ 0 \\ -100 \end{pmatrix} \text{ mA}$$



$$V_1 = 10V$$

$$2) \frac{V_2 - 10}{10000} + \frac{V_2 - V_3}{30000} + \frac{V_2}{30000} = 0$$

$$\frac{1}{10000} V_2 \left(\frac{1}{10} + \frac{1}{3} + \frac{1}{30} \right) = \frac{1}{10000} \left(\frac{V_3}{3} + 1 \right)$$

$$V_2 \left(\frac{3 + 10 + 1}{30} \right) = \frac{V_3 + 3}{3}$$

$$V_2 = (V_3 + 3) \frac{10}{14}$$

$$V_2 = \frac{5}{7} V_3 + \frac{15}{7}$$

$$3) \frac{V_3 - 10}{20} + \frac{V_3 - V_2}{3} + \frac{V_3}{40} = 0$$

$$V_3 \left(\frac{1}{20} + \frac{1}{3} + \frac{1}{40} \right) - \frac{5}{21} V_3 - \frac{15}{21} - \frac{1}{2} = 0$$

$$V_3 \left(\frac{1}{20} + \frac{1}{3} + \frac{1}{40} - \frac{5}{21} \right) - \frac{5}{7} - \frac{1}{2} = 0$$

$$V_3 \left(\frac{42 + 280 + 21 - 200}{840} \right) = \frac{+10 + 7}{14}$$

$$V_3 \left(\frac{143}{60} \right) = 17$$

$$V_3 = 7.13286V$$

$$V_2 = 7.23776$$

$$V_x = V_2 - V_3$$

$$V_x = 0.105V$$

By KCL at ①

$$-I_x + \frac{V_1 - V_2}{10k\Omega} + \frac{V_1 - V_3}{20000} = 0$$

$$I_x = \frac{10 - 7.23776}{10000} + \frac{10 - 7.13286}{20000}$$

$$= \frac{5.52448 + 2.86714}{20000}$$

$$I_x = 0.419 \times 10^{-3} A$$

$$I_x = 0.419 \text{ mA}$$

$$P = VI$$

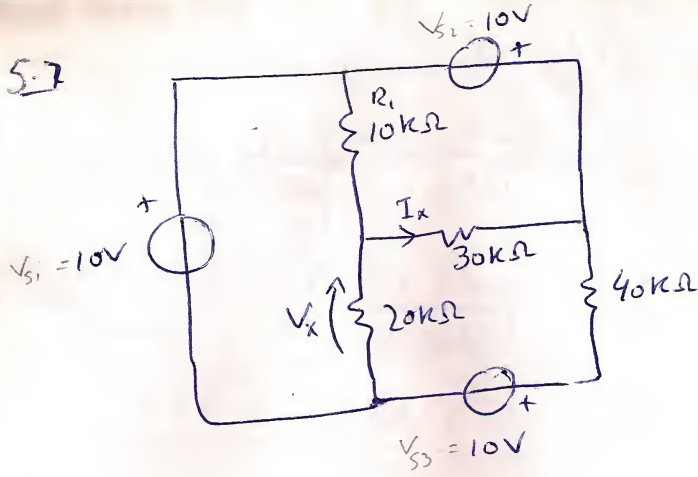
$$P = \frac{V^2}{R_2}$$

$$P = \frac{(V_1 - V_3)^2}{20000}$$

$$P = \frac{8.22049}{20} \times 10^{-3}$$

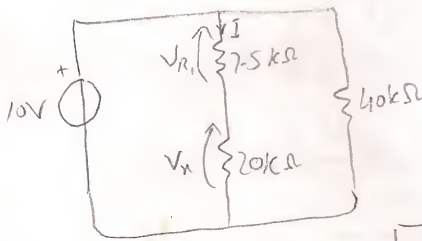
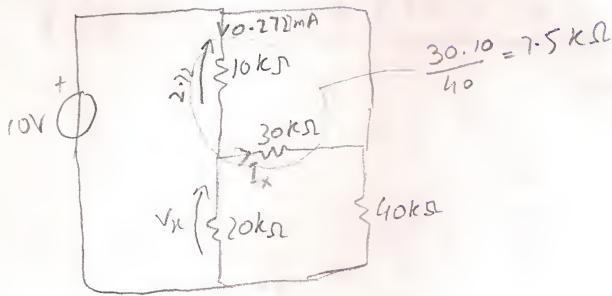
$$P = 0.411 \text{ mW}$$

5.7



By superposition

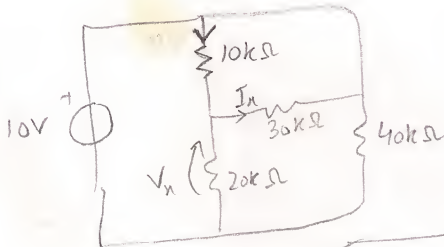
-Contribution of V_{S1}



$$V_x = \frac{20}{7.5 + 20} \times 10 \Rightarrow V'_x = 7.27V$$

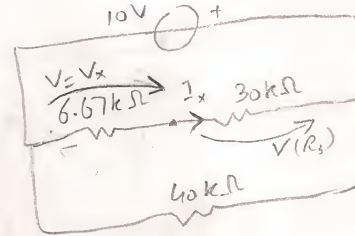
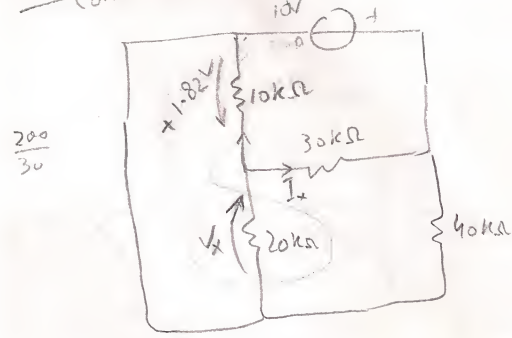
$$V(7.5) = \frac{7.5}{27.5} \times 10 \Rightarrow V(7.5) = 2.7272V$$

$$I(7.5) = \frac{2.72}{7.5} \Rightarrow I(7.5) = 0.36mA$$



$$I_x = -\frac{10}{10 + 30} \times 0.36m \Rightarrow I'_x = -0.09mA$$

-Contribution of V_{S2}



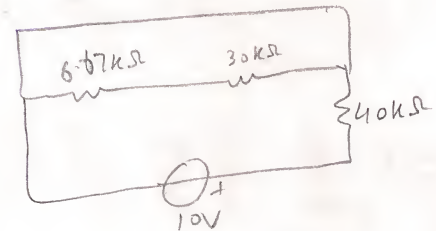
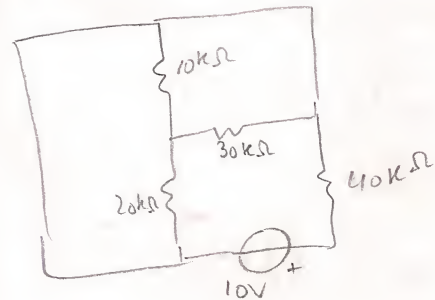
$$V = V_x = \frac{6.67}{30 + 6.67} \times 10 \Rightarrow V'_x = 1.82V$$

$$V(R_3) = \frac{30}{36.67} \times 10 \Rightarrow V(R_3) = 8.18V$$

$$I_x = -\frac{V(R_3)}{R_3}$$

$$I'_x = -0.2727mA$$

-Contribution of V_{S3}



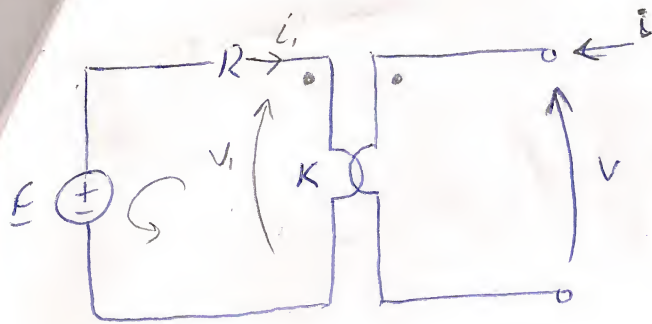
$$I_x = V_x = 0$$

$$V_x = 7.27 + 1.82 = 9.09V$$

$$I_x = -0.09 \times 10^{-3} + -0.2727 \times 10^{-3} = -0.36mA$$

$$P(R_1) = R_1 I^2 = 10000 (0.272 \pm 0.182)^2$$

$$P(R) = 82.2 \mu W$$



$$V_1 = KV$$

$$i_1 = -\frac{1}{K} i_2$$

By KVL

$$V_1 + i_1 R = E$$

$$KV - \frac{R}{K} i_2 = E$$

$$V = \frac{E}{K} + \frac{R}{K^2} i_2$$

Hence $R_{eq} = \frac{R}{K^2} \Omega$ ✓
 $V_{eq} = \frac{E}{K} V$ ✓

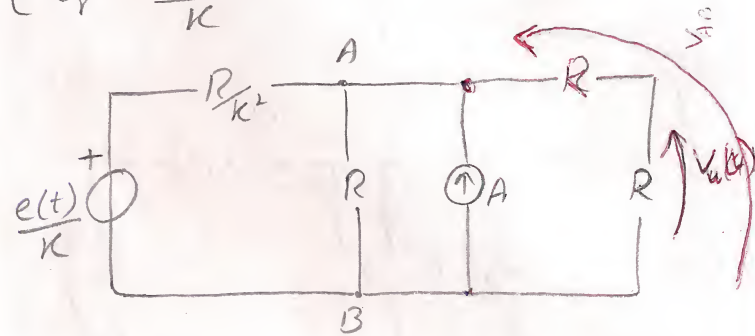
By KVL

$$V_1 + i_1 R = e(t)$$

$$KV_2 - \frac{R}{K} i_2 = e(t)$$

$$V_2 = \frac{e(t)}{K} + \frac{R}{K^2} i_2$$

$$\begin{cases} R_{eq} = \frac{R}{K^2} \\ V_{eq} = \frac{e(t)}{K} \end{cases}$$



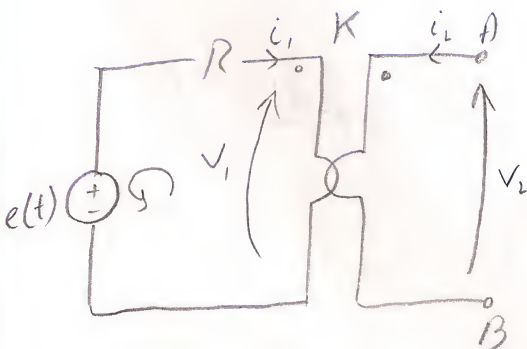
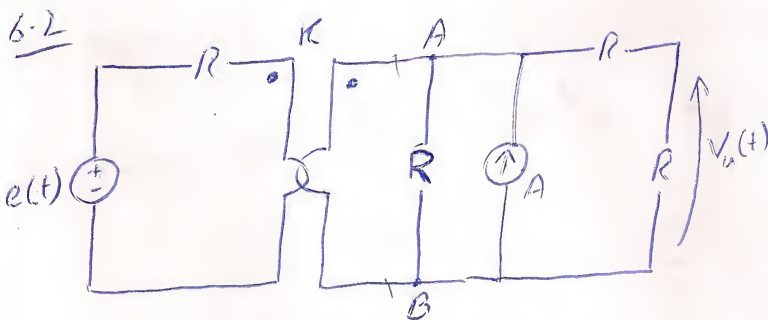
By Millmann

$$V_n(t) = \frac{V_{AB}}{2} = \frac{\frac{e(t)}{K} \cdot \frac{K^2}{R} + A}{\frac{1}{R/K^2} + \frac{1}{R} + \frac{1}{R+R}} \cdot \frac{1}{2}$$

$$= \frac{K \cdot e(t) + RA}{\frac{K^2}{R} + \frac{1}{R} + \frac{1}{2R}} \cdot \frac{1}{2R}$$

$$= \frac{K \cdot e(t) + RA}{\frac{2K^2 + 2 + 1}{2R}} \cdot \frac{1}{2R}$$

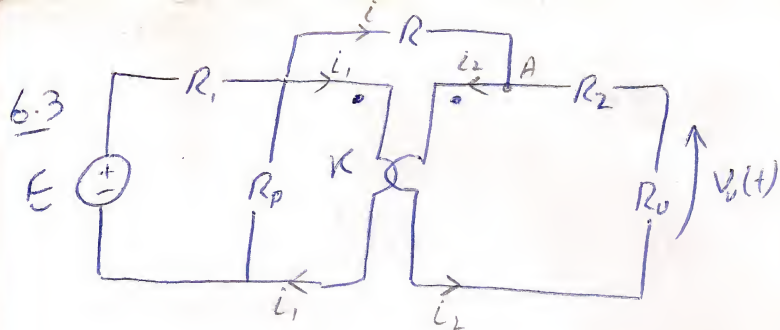
$$V_n(t) = \frac{K e(t) + RA}{2K^2 + 3} V$$



$$\begin{cases} V_1 = KV_2 \\ i_1 = -\frac{1}{K} i_2 \end{cases}$$

$$V_n(t) = \frac{R}{R+R} V_{AB} = \frac{V_{AB}}{2} = \frac{V_{AB}}{2} \cdot \frac{1}{2}$$

↓
Millmann

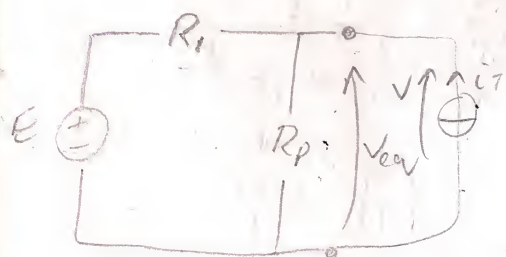
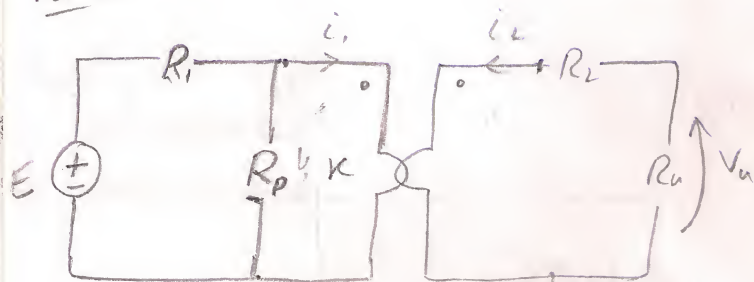


KCL @ A

$$-i + i_1 - i_2 = 0$$

$$i = 0$$

Hence



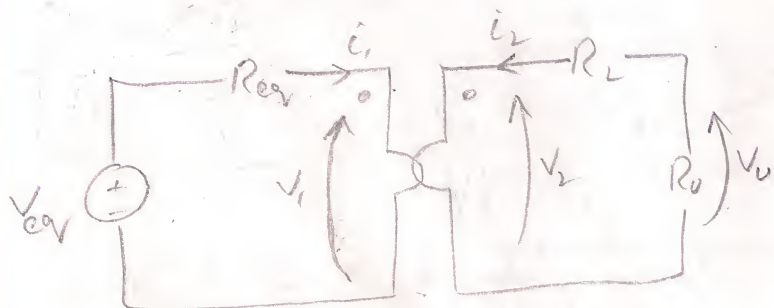
$$V = \frac{E + i_1 R_1}{\frac{1}{R_1} + \frac{1}{R_p}}$$

$$V = \frac{E + R_1 i_1}{\frac{R_1 + R_p}{R_1 R_p}}$$

$$V = \frac{R_p E}{R_1 + R_p} + \frac{R_1 R_p i_1}{R_1 + R_p}$$

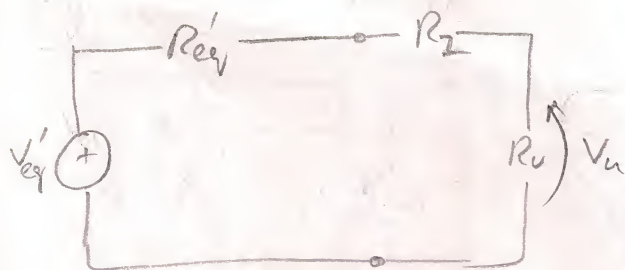
$$R_{eq} = \frac{R_1 R_p}{R_1 + R_p}$$

$$V_{eq} = \frac{R_p}{R_1 + R_p} E$$



$$V_{eq}' = \frac{V_{eq}}{K} = \frac{R_p E}{(R_1 + R_p) K}$$

$$R_{eq}' = \frac{R_{eq}}{K^2} = \frac{R_1 R_p}{K^2 (R_1 + R_p)}$$



$$V_u = \frac{R_u}{R_{eq}' + R_2 + R_u} \times V_{eq}'$$

$$= \frac{R_u}{\frac{R_1 R_p}{K^2 (R_1 + R_p)} + R_2 + R_u} \cdot \frac{R_p E}{(R_1 + R_p) K}$$

$$= \frac{K^2 (R_1 + R_p) R_u R_p E}{K (R_1 + R_p) [R_1 R_p + K^2 (R_1 + R_p) (R_2 + R_u)]}$$

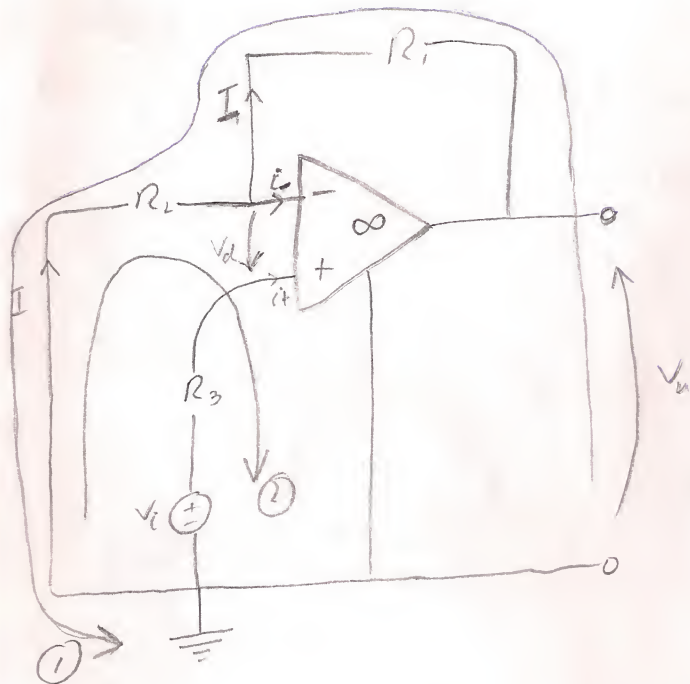
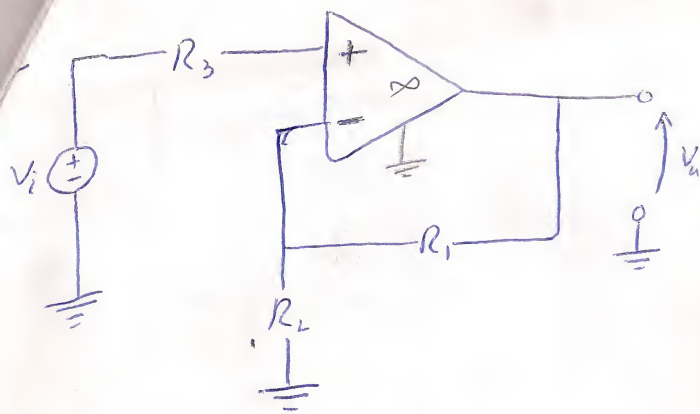
$$V_u = E \cdot \frac{R_u R_p K}{R_1 R_p + K^2 (R_p + R_1) (R_2 + R_u)}$$

6.4 $\frac{V_i - V_o}{R_3} = 0 \Rightarrow V_i = V_o$
 $V_2 = V_1 = V_i$

$$\frac{V_i}{R_2} + \frac{V_i - V_u}{R_1} = 0$$

$$V_i \left(\frac{R_1 + R_2}{R_1 R_2} \right) = \frac{V_u}{R_1}$$

$$V_u = V_i \left(\frac{R_1 + R_2}{R_2} \right)$$



KVL

$$1) V_u + IR_1 + IR_2 = 0 \quad \text{--- (1)}$$

$$2) -IR_2 + V_d + R_3 i_1 - V_i = 0$$

$$IR_2 = -V_i$$

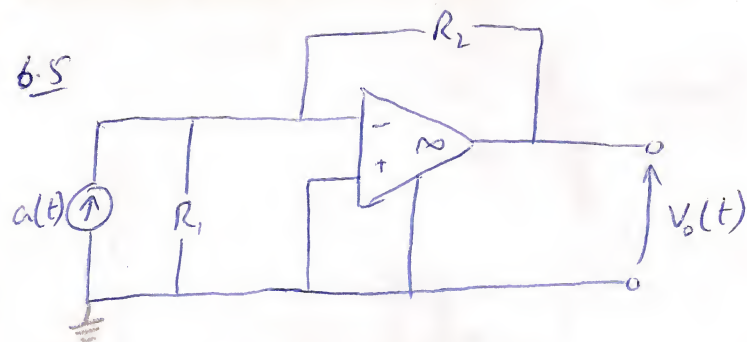
$$I = \frac{-V_i}{R_2} \text{ put in (1)}$$

$$V_u + \frac{V_i}{R_2} R_1 - V_i \frac{R_2}{R_2} = 0$$

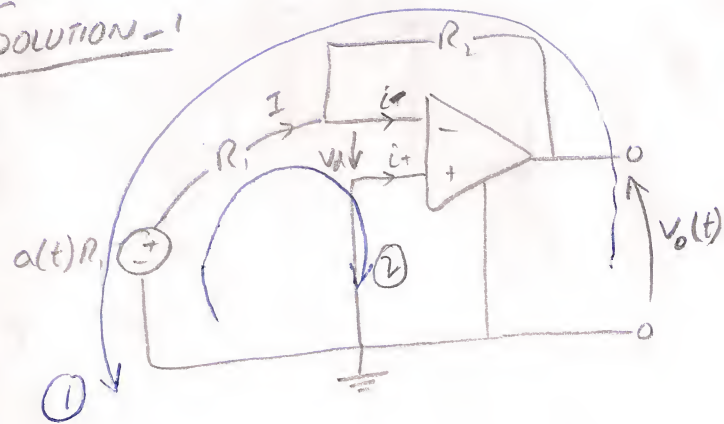
$$V_u = V_i \left(1 + \frac{R_1}{R_2} \right)$$

$$V_u = V_i \left(\frac{R_1 + R_2}{R_2} \right) \underline{\underline{R_2}}$$

6.5



Solution - 1



KVL

$$1) V_o + I(R_1 + R_2) - aR_1 = 0$$

$$2) aR_1 - IR_1 + V_d = 0$$

$$aR_1 = IR_1$$

$$a = I \text{ put in (1)}$$

$$V_o + a(R_1 + R_2 - R_1) = 0$$

$$V_o(t) = -a(t)R_2$$

$$V_o(t) = -0.5 \sin(500t) \cdot 10 \cdot 10^3 \cdot 10$$

$$V_o(t) = -5 \sin(500t) \text{ V}$$

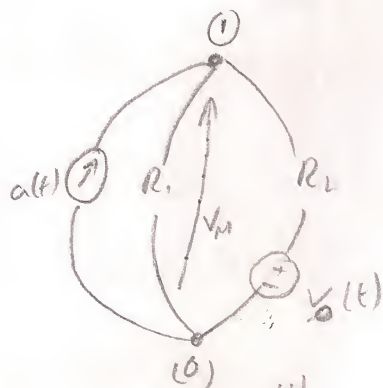
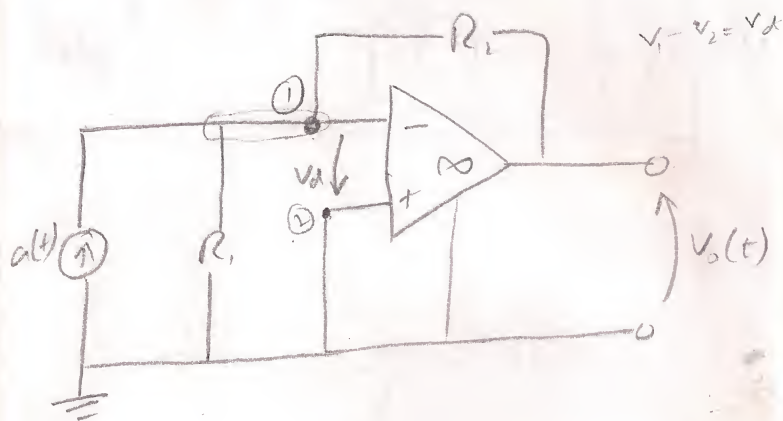
$$V_i = V_o = 0$$

$$-a(t) + \frac{V_i}{R_1} + \frac{V_i - V_o(t)}{R_2} = 0$$

$$-\frac{V_o(t)}{R_2} = a(t)$$

$$V_o(t) = -a(t)R_2$$

Solution-2 By Millmann



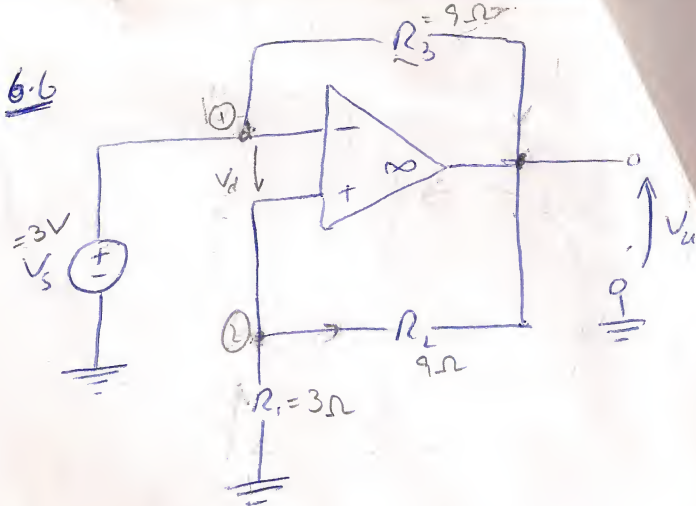
$$V_M = \frac{a(t) + \frac{V_o(t)}{R_2}}{\frac{1}{R_1} + \frac{1}{R_2}}$$

$$V_M = -V_d = 0$$

$$0 = a(t) + \frac{V_o(t)}{R_2}$$

$$V_o(t) = -a(t) R_2$$

6.6



$$V_1 = V_2 = V_5 = 3V$$

② KCL

$$\frac{V_2}{R_1} + \frac{V_2 - V_n}{R_2} = 0$$

$$\frac{3}{3} + \frac{3 - V_n}{9} = 0$$

$$\frac{V_n}{9} = \frac{4}{3}$$

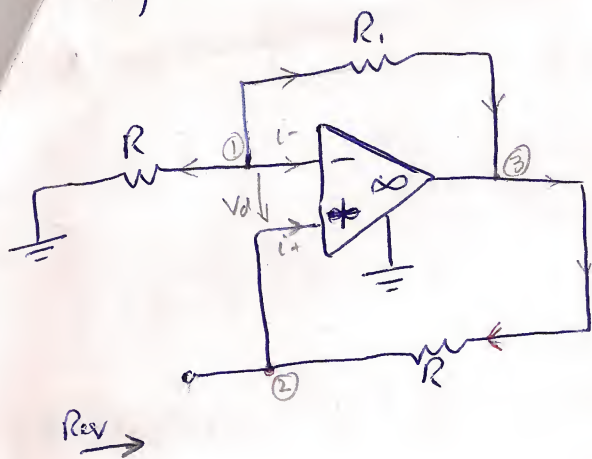
$$V_n = 12V \text{ Ans}$$

$$\frac{V_n}{R_2} = \frac{V_2}{R_1} + \frac{V_2}{R_2}$$

$$\frac{V_n}{R_2} = \frac{R_2 V_2 + R_1 V_2}{R_1 R_2}$$

$$V_n = V_2 \left(\frac{R_1 + R_2}{R_1} \right) V \text{ Ans}$$

Compute equivalent resistance



$$\textcircled{1} \quad \frac{V_1}{R} + \frac{V_1 - V_3}{R_1} = 0$$

$$V_1 \left(\frac{1}{R} + \frac{1}{R_1} \right) = \frac{V_3}{R_1}$$

$$V_1 \left(\frac{R + R_1}{R R_1} \right) = \frac{V_3}{R_1}$$

$$V_1 \left(\frac{R + R_1}{R} \right) = V_3 \quad \text{--- (1)}$$

$$\textcircled{2} \quad \frac{V_2}{R_{eq}} - \frac{V_2 - V_3}{R} = 0$$

entering current

$$V_1 = V_2$$

$$V_1 \left(\frac{1}{R_{eq}} - \frac{1}{R} \right) = -\frac{V_3}{R}$$

$$V_1 \left(\frac{R - R_{eq}}{R_{eq} R} \right) = -\frac{V_3}{R}$$

$$-V_1 \left(\frac{R - R_{eq}}{R_{eq}} \right) = V_3 \quad \text{--- (2)}$$

comp (1) & (2)

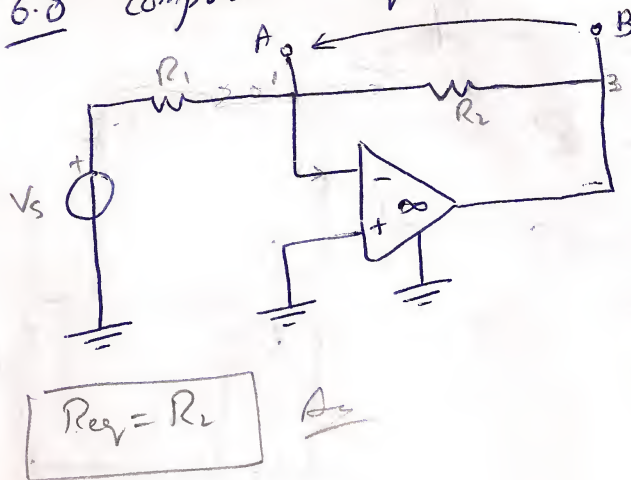
$$\frac{R + R_1}{R} = -\left(\frac{R - R_{eq}}{R_{eq}} \right)$$

$$\frac{R + R_1}{R} = -\frac{R}{R_{eq}} + 1$$

$$\frac{R}{R_{eq}} = -\frac{R + R_1}{R} + 1 \Rightarrow \frac{R}{R_{eq}} = \frac{-R - R_1 + R}{R}$$

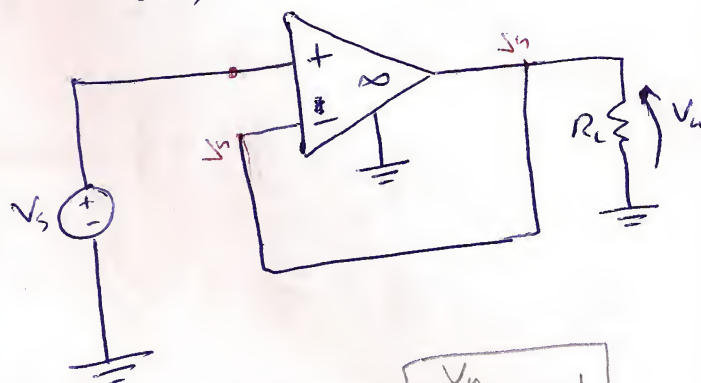
$$\frac{R}{R_{eq}} = -\frac{R_1}{R} \Rightarrow \boxed{R_{eq} = -\frac{R^2}{R_1} \Omega} \quad \underline{\text{Ans}}$$

6.8 compute R_{eq} at AB



$$\boxed{R_{eq} = R_2} \quad \underline{\text{Ans}}$$

6.9 Derive $\frac{V_u}{V_s} = ?$
 $P(R_L) = ?$, $P(V_s) = ?$



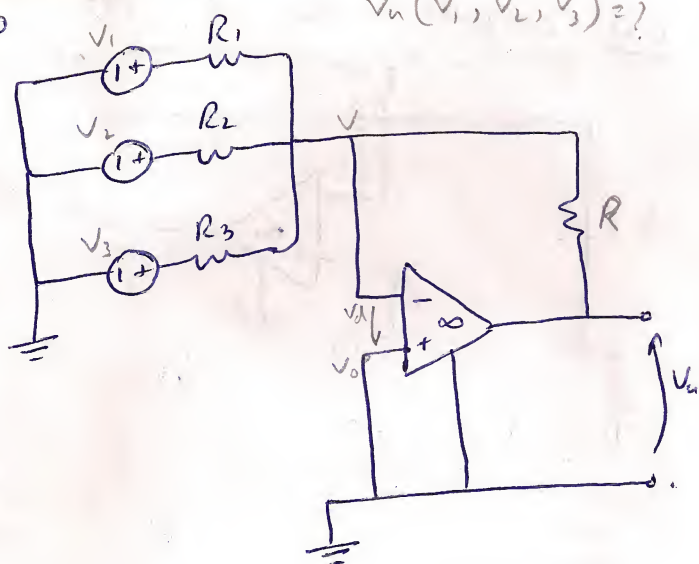
a) $V_u = V_s \Rightarrow \boxed{\frac{V_u}{V_s} = 1}$

b) Power generated by the source will be the multiplication of source voltage and the current through it.

c) $P(R_L) = \frac{V_u^2}{R_L} = \frac{V_s^2}{R_L} = \frac{(3V)^2}{3 \times 10^3 \Omega}$
 $= \frac{9}{3 \times 10^3}$

$$\boxed{P(R_L) = 3 \text{ mW}} \quad \underline{\text{Ans}}$$

6.10



$$V_u(V_1, V_2, V_3) = ?$$

$$V = V_0$$

$$V \left(\frac{1}{R} + \frac{1}{R_f} \right) = \frac{V_2}{R}$$

$$V \left(\frac{R + R_f}{R R_f} \right) = \frac{V_2}{R}$$

$$V = V_2 \left(\frac{R_f}{R + R_f} \right) \text{ put in ①}$$

$$\textcircled{1} \quad V \left(\frac{1}{R} + \frac{1}{R_f} \right) - \frac{V_1}{R} = \frac{V_u}{R_f}$$

$$V_2 \left(\frac{R_f}{R + R_f} \right) \left(\frac{R + R_f}{R R_f} \right) - \frac{V_1}{R} = \frac{V_u}{R_f}$$

$$\frac{V_2}{R} - \frac{V_1}{R} = \frac{V_u}{R_f}$$

$$V_u = \frac{R_f}{R} (V_2 - V_1) \quad \underline{\underline{Ans}}$$

$$V = V_0 \Rightarrow (V_0 = 0)$$

$$V = 0$$

$$\textcircled{1} \quad \frac{V - V_1}{R_1} + \frac{V - V_2}{R_2} + \frac{V - V_3}{R_3} + \frac{V - V_u}{R} = 0$$

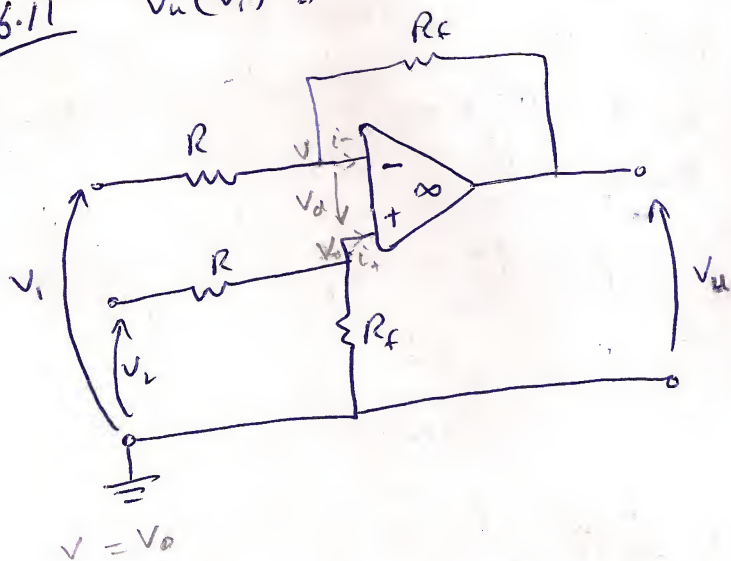
$$-\frac{V_1}{R_1} - \frac{V_2}{R_2} - \frac{V_3}{R_3} - \frac{V_u}{R} = 0$$

$$\frac{V_u}{R} = - \left(\frac{V_1}{R_1} + \frac{V_2}{R_2} + \frac{V_3}{R_3} \right)$$

$$V_u = - \left(V_1 \frac{R}{R_1} + V_2 \frac{R}{R_2} + V_3 \frac{R}{R_3} \right) \underline{\underline{Ans}}$$

6.11

$$V_u(V_1, V_2) = ?$$

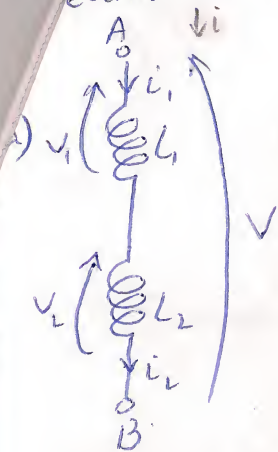


$$V = V_0$$

$$\textcircled{1} \quad \frac{V - V_1}{R} + \frac{V - V_u}{R_f} = 0$$

$$\textcircled{2} \quad \frac{V_0 - V_2}{R} + \frac{V_0}{R_f} = 0$$

determine characteristic equations



$$V_1 = L_1 \frac{di_1}{dt}$$

$$V_2 = L_2 \frac{di_2}{dt}$$

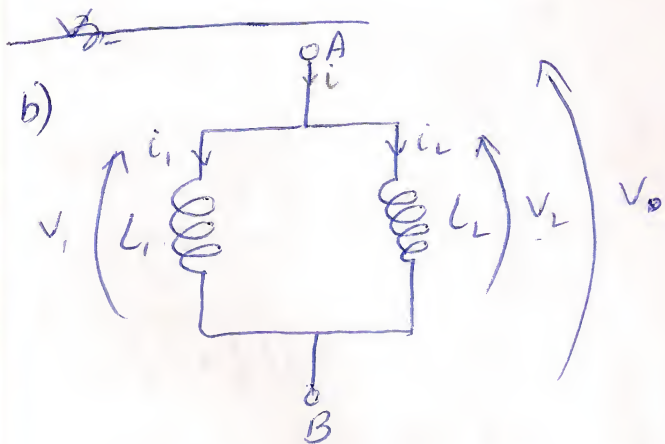
$$i = i_1 = i_2$$

$$V = V_1 + V_2$$

$$V = L_1 \frac{di}{dt} + L_2 \frac{di}{dt}$$

$$V = \frac{di}{dt} (L_1 + L_2)$$

$$(i_1 = i_2 = i)$$



$$i = i_1 + i_2$$

$$V = V_1 = V_2$$

$$V_1 = L_1 \frac{di_1}{dt}$$

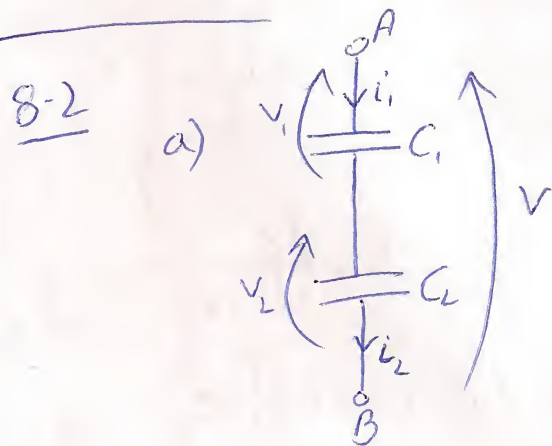
$$V_2 = L_2 \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}$$

$$\frac{di}{dt} = \frac{V}{L_1} + \frac{V}{L_2}$$

$$\frac{di}{dt} = V \left(\frac{L_1 + L_2}{L_1 L_2} \right)$$

$$V = \left(\frac{L_1 L_2}{L_1 + L_2} \right) \frac{di}{dt}$$



$$i = C_1 \frac{dv_1}{dt}$$

$$i = i_1 = i_2$$

$$i_2 = C_2 \frac{dv_2}{dt}$$

$$V = V_1 + V_2$$

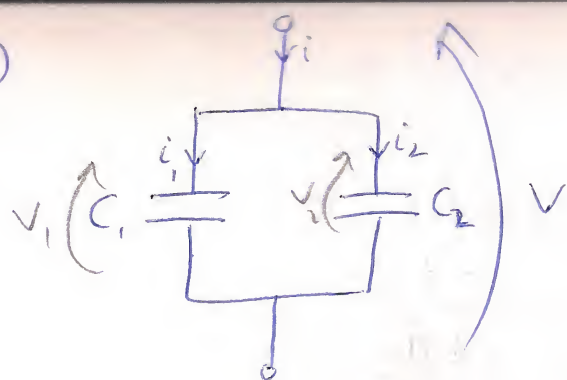
$$\frac{dv}{dt} = \frac{dv_1}{dt} + \frac{dv_2}{dt}$$

$$\frac{dv}{dt} = \frac{i}{C_1} + \frac{i}{C_2}$$

$$\frac{dv}{dt} = i \left(\frac{C_1 + C_2}{C_1 C_2} \right)$$

$$i = \frac{C_1 C_2}{C_1 + C_2} \frac{dv}{dt}$$

b)



$$i_1 = C_1 \frac{dv_1}{dt}$$

$$i = i_1 + i_2$$

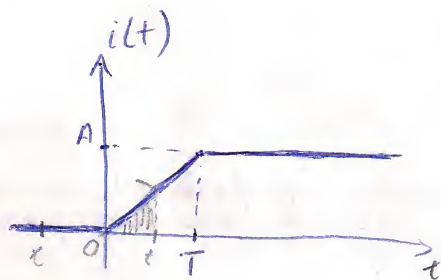
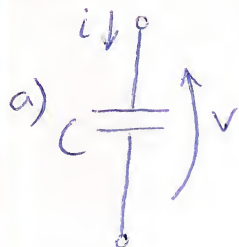
$$i_2 = C_2 \frac{dv_2}{dt}$$

$$V = V_1 = V_2$$

$$i = C_1 \frac{dv}{dt} + C_2 \frac{dv}{dt}$$

$$\boxed{i = \frac{dv}{dt} (C_1 + C_2)} \quad R_2$$

8.3 Determine the voltage



$$i(t) = C \frac{dv(t)}{dt}$$

$$v(t) = \int \frac{i(t)}{C} dt$$

$$-v(t) = 0 \quad t < 0$$

$$-v(t) = \int_0^t \frac{i(t)}{C} dt \quad 0 \leq t < T$$

$$= \frac{1}{C} \int_0^t i(t) dt$$

$$(0,0), (T,A)$$

$$(0-A) = m(0-T)$$

$$m = \frac{A}{T}$$

$$i(t) - 0 = \frac{A}{T}(t - 0)$$

$$\boxed{i(t) = \frac{A}{T} t}$$

$$i(t) = \frac{A}{T} t$$

$$= \frac{A}{TC} \int_0^t t \cdot dt$$

$$= \frac{A}{TC} \cdot \frac{t^2}{2}$$

$$= \frac{A}{CT} \cdot \frac{t^2}{2}$$

$$-v(t) = \int_T^{+\infty} \frac{i(t)}{C} dt \quad t \geq T$$

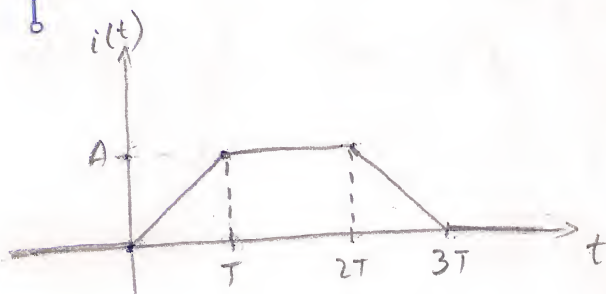
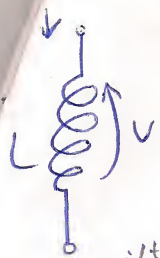
$$= \int_0^T \frac{i(t)}{C} dt + \int_T^t \frac{i(t)}{C} dt$$

$$= \int_0^T \frac{A}{TC} t dt + \int_T^t \frac{A}{C} dt$$

$$= \frac{A}{TC} \cdot \frac{T^2}{2} + \frac{A}{C} (t - T)$$

$$v(t) = \frac{AT}{2C} + \frac{A}{C} (t - T)$$

$$v(t) = \begin{cases} 0 & t < 0 \\ \frac{A}{CT} \cdot \frac{t^2}{2} & 0 \leq t < T \\ \frac{AT}{2C} + \frac{A}{C} (t - T) & t \geq T \end{cases}$$



$$v(t) = L \frac{di(t)}{dt}$$

$$\boxed{v(t) = 0 \text{ V}}$$

$$- 0 \leq t < T$$

$$i(t) = \frac{A}{T} t$$

$$v(t) = \frac{A}{T} L \frac{d}{dt} t$$

$$\boxed{v(t) = L \frac{A}{T} \text{ V}}$$

$$- T \leq t < 2T$$

$$i(t) = A$$

$$v(t) = L \frac{d}{dt} (A)$$

$$\boxed{v(t) = 0}$$

$$- 2T \leq t \leq 3T$$

$$i(t) = -\frac{A}{T} t + 3A \text{ (-ve slope)}$$

$$v(t) = L \frac{d}{dt} \left(-\frac{A}{T} t \right)$$

$$\boxed{v(t) = -\frac{A}{T} L \text{ V}}$$

$$t < 0$$

$$\begin{aligned} (0,0) \quad (T,A) \\ (0-A) &= m(0-T) \\ m &= \frac{A}{T} \\ i(t)-0 &= \frac{A}{T}(t-0) \\ \boxed{i(t) &= \frac{A}{T} t} \end{aligned}$$

$$m(x_1 - x_2) = -(y_1 - y_2)$$

$$\begin{aligned} m(2T-3T) &= A-0 \\ m &= -\frac{A}{T} \end{aligned}$$

$$\begin{aligned} (2T,A) \quad (3T,0) \\ A &= m(-T) \\ m &= -\frac{A}{T} \end{aligned}$$

$$i(t) = -\frac{A}{T} t + 3A$$

$$i(t) - A = -\frac{A}{T}(t - 2T)$$

$$i(t) = -\frac{A}{T} t + 2T \frac{A}{T} + A$$

$$\boxed{i(t) = -\frac{A}{T} t + 3A}$$

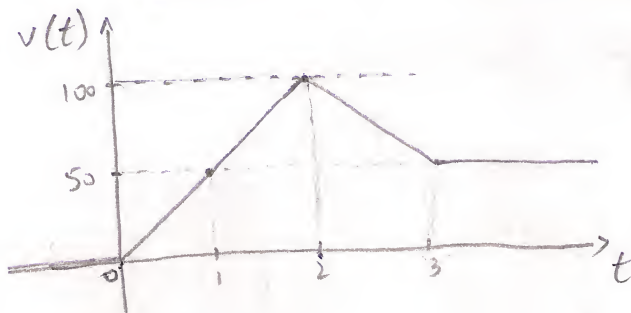
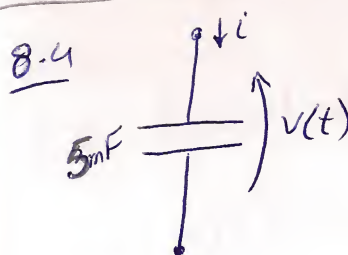
$$-3T \leq t$$

$$i(t) = 0$$

$$\boxed{v(t) = 0 \text{ V}}$$

Hence

$$v(t) = \begin{cases} 0 \text{ V} & t < 0 \\ L \frac{A}{T} \text{ V} & 0 \leq t < T \\ 0 \text{ V} & T \leq t < 2T \\ -L \frac{A}{T} \text{ V} & 2T \leq t < 3T \\ 0 \text{ V} & t \geq 3T \end{cases}$$



$$i(t) = C \frac{dv(t)}{dt}$$

$$- t < 0, v(t) = 0$$

$$\boxed{i(t) = 0 \text{ A}}$$

$$- 0 \leq t < 2$$

$$v(t) = 50t$$

$$i(t) = 5 \times 50 \times \frac{d}{dt} t$$

$$\boxed{i(t) = 0.25 \text{ A}}$$

$$- 2 \leq t < 3$$

$$(100, 2), (50, 3)$$

$$(100 - 50) = m(2 - 3)$$

$$m = -50$$

$$(V - 100) = -50(t - 2) \quad ?$$

$$V(t) = -50t + 100 + 100$$

$$V(t) = -50t + 200$$

$$i(t) = 5 \times 10^{-3} \frac{d}{dt} (-50t + 200)$$

$$= -5.50 \times 10^{-3} \text{ A}$$

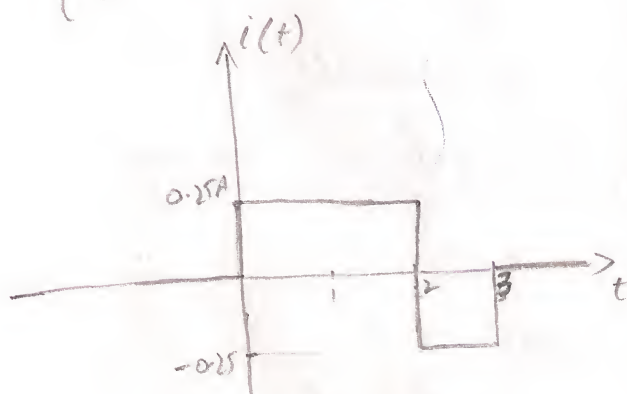
$$i(t) = -0.25 \text{ A}$$

$$- t \geq 3$$

$$V(t) = 50 \text{ V}$$

$$i(t) = 0 \text{ A}$$

$$i(t) = \begin{cases} 0 \text{ A} & t < 0 \\ 0.25 \text{ A} & 0 \leq t < 2 \\ -0.25 \text{ A} & 2 \leq t < 3 \\ 0 & t \geq 3 \end{cases}$$



$$P(t) = V(t)i(t)$$

$$P(t) = V(t) C \frac{dV(t)}{dt}$$

$$- t < 0$$

$$V(t) = 0$$

$$P(t) = 0 \text{ W}$$

$$- 0 \leq t < 2$$

$$V(t) = 50t$$

$$P(t) = 50t \times 5 \text{ m} \times 50$$

$$P(t) = 12.5t \text{ W}$$

$$- 2 \leq t < 3$$

$$(100, 2), (50, 3)$$

$$(100 - 50) = m(2 - 3) \Rightarrow m = -50$$

$$V(t) - 100 = -50(t - 2)$$

$$V(t) = -50t + 200$$

$$P(t) = 5 \text{ m} (-50t + 200) \frac{d}{dt} (-50t + 200)$$

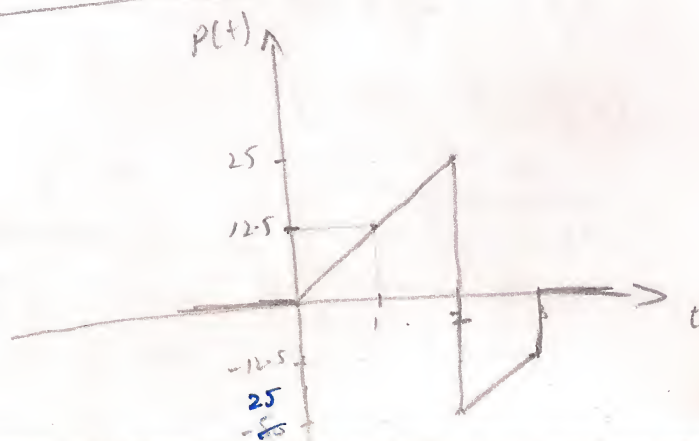
$$= 250 \times 10^{-3} (t - 4) \times -50$$

$$P(t) = 12.5(t - 4) \text{ W}$$

$$- t \geq 3$$

$$V(t) = 50 \text{ V}$$

$$P(t) = 0 \text{ W}$$



$$\frac{1}{2} C \dot{V}(t)$$

$$t < 0$$

$$V(t) = 0$$

$$U = 0 \text{ J}$$

$$- 0 \leq t < 2$$

$$V(t) = 50t$$

$$U = \frac{1}{2} 5m \cdot 2500t^2$$

$$U = 6.25t^2 \text{ J}$$

$$- 2 \leq t < 3$$

$$(100, 2) \quad (50, 3)$$

$$(100 - 50) = m(2 - 3) \Rightarrow m = -50$$

$$V(t) - 100 = -50(t - 2)$$

$$V(t) = -50t + 200 \text{ V}$$

$$U(t) = \frac{1}{2} 5m \cdot 2500 (t - 4)^2$$

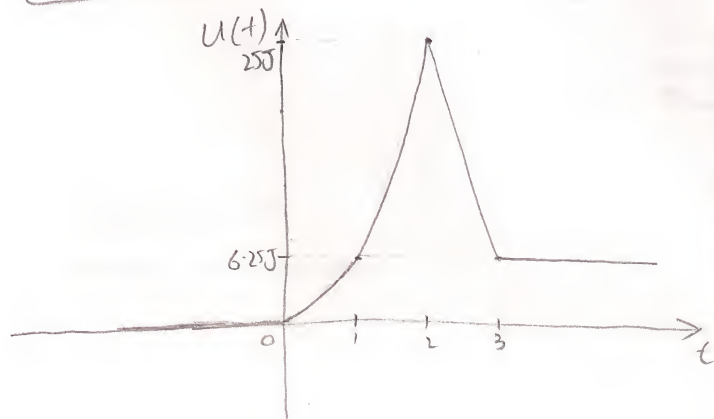
$$U(t) = 6.25(t - 4)^2 \text{ J}$$

$$t \geq 3$$

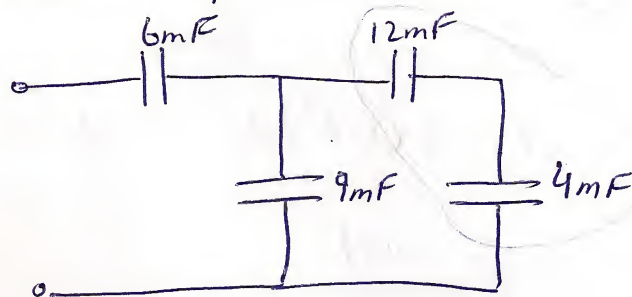
$$V(t) = 50 \text{ V}$$

$$U(t) = \frac{1}{2} 5m \cdot 2500$$

$$U(t) = 6.25 \text{ J}$$



8.5 Derive equivalent capacitance.

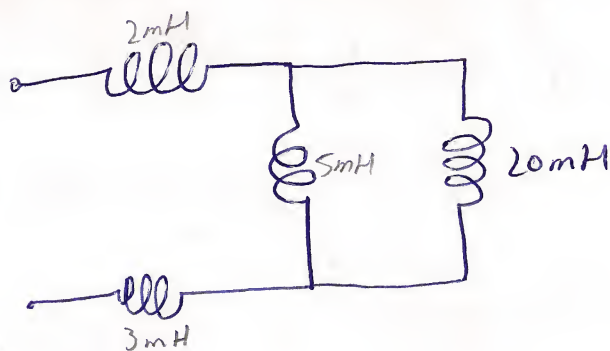


$$\frac{12 \cdot 4}{16} = 3 \text{ mF}$$

$$3 + 9 = 12 \text{ mF}$$

$$C_{eq} = \frac{12 \cdot 6}{18} = 4 \text{ mF} \underline{\underline{Ans}}$$

8.6 Equivalent inductance



$$\frac{20 \cdot 5}{25} = 4 \text{ mH}$$

$$L_{eq} = 4 + 2 + 3 = 9 \text{ mH} \underline{\underline{Ans}}$$

8.7 $L = 10 \text{ mH}$

$i(t) = 0 \quad t < 0$

$i(t) = 5(1 - e^{-t}) \text{ A} \quad t \geq 0$

Voltage $v(t)$

- $t < 0$

$i(t) = 0$

$v(t) = L \frac{di(t)}{dt}$

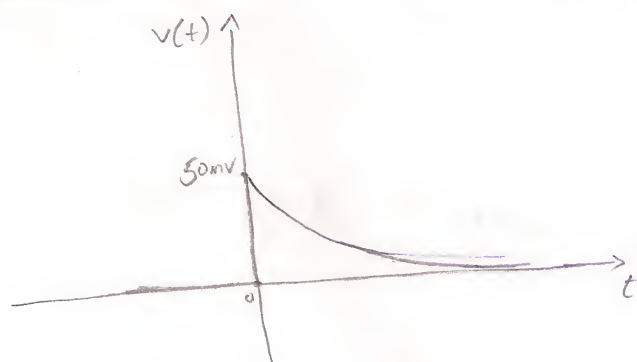
$v(t) = 0$

- $t \geq 0$

$i(t) = 5(1 - e^{-t}) \text{ A}$

$v(t) = 10 \times 10^{-3} \frac{d}{dt} (5 - 5e^{-t})$
 $= 10 \times 10^{-3} 5e^{-t}$

$v(t) = 50e^{-t} \text{ mV}$



Power $P(t) = v(t) i(t)$

- $t < 0$

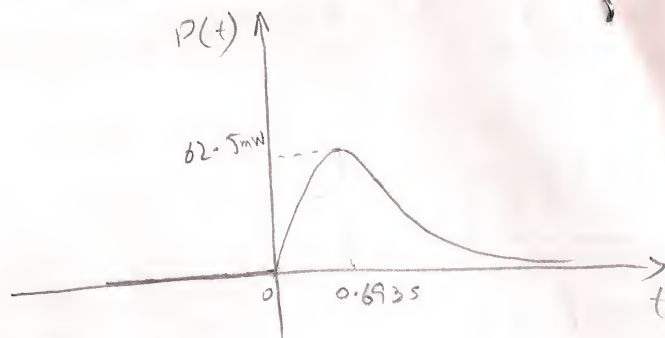
$i(t) = 0, \quad v(t) = 0$

$P(t) = 0$

- $t \geq 0$

$i(t) = 5(1 - e^{-t}) \text{ A} \quad v(t) = 50e^{-t} \text{ mV}$

$P(t) = 250(1 - e^{-t})e^{-t} \text{ mW}$



Energy $(U = \frac{1}{2} L i^2)$

- $t < 0$

$i(t) = 0$

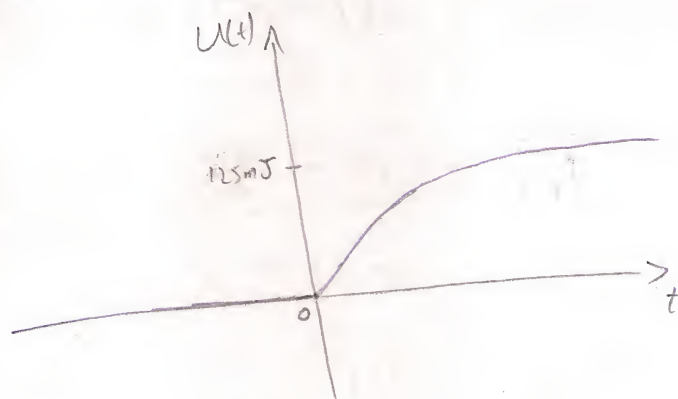
$U(t) = 0 \text{ J}$

- $t \geq 0$

$i(t) = 5(1 - e^{-t}) \text{ A}$

$U = \frac{1}{2} 10 \times 10^{-3} 25 (1 - e^{-t})^2 \text{ J}$

$U = 125 (1 - e^{-t})^2 \text{ mJ}$



$$\tau_c = R_{eq}C$$

$$\tau_L = \frac{L}{R_{eq}}$$

$$v_c(t) = [v(0) - v_{\infty}] e^{-\frac{t-t_0}{\tau}} + v_{\infty} \quad (t \geq 0)$$

$$i_L(t) = [i_L(0) - i_{L\infty}] e^{-\frac{t-t_0}{\tau}} + i_{L\infty} \quad (t \geq 0)$$

$$(V_0 - v) e^{-\frac{(t-t_0)}{\tau}} + v$$

Chapter 9

First order circuits

Exercise 9.1

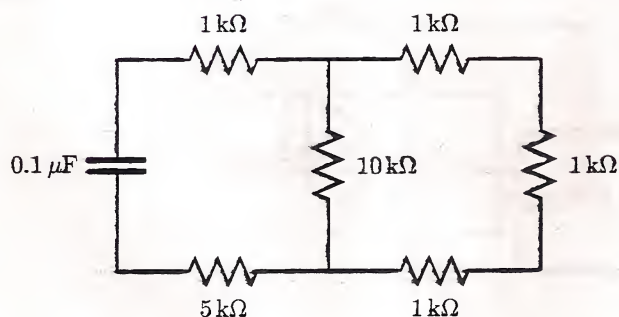
Find the waveform $v(t)$ that satisfies the following differential equation:

$$\frac{dv(t)}{dt} + 10v(t) = 0$$

with $v(0) = 5V$.

Exercise 9.2

Find the time constant of the circuit.



Exercise 9.3

Find the time constant τ of the circuit.

Find the waveform $v(t)$ that satisfies the following differential equation.

$$\frac{dv(t)}{dt} + 10v(t) = 0$$

Ans $\frac{dx}{dt} + \frac{dx}{\tau} = \frac{G_{eq}}{\tau}$

$$\frac{dv(t)}{dt} = -10v(t)$$

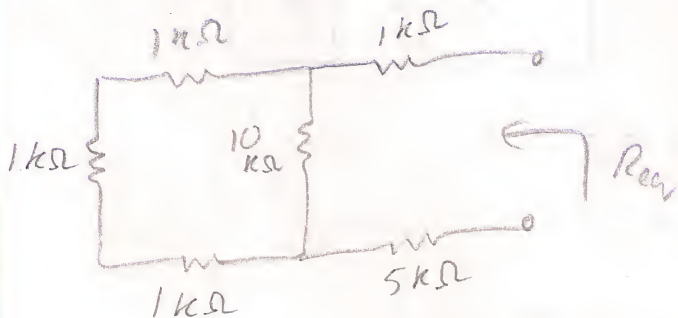
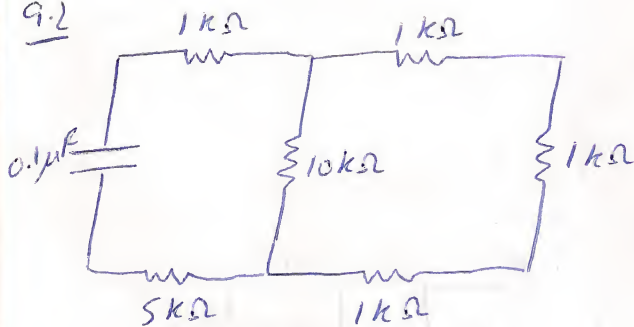
$$\int \frac{dv(t)}{v(t)} = \int -10 dt$$

$$\ln v(t) = -10t$$

$$\ln \frac{v(t)}{v(0)} = -10t \Rightarrow \frac{v(t)}{v(0)} = e^{-10t}$$

$$v(t) = 5e^{-10t}$$

9.2



$$R_{eq} = 3 \parallel 10 + 5$$

$$= \frac{30}{13} + 5$$

$$R_{eq} = 8.30 k\Omega$$

$$\tau = R_{eq}C$$

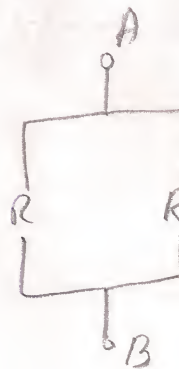
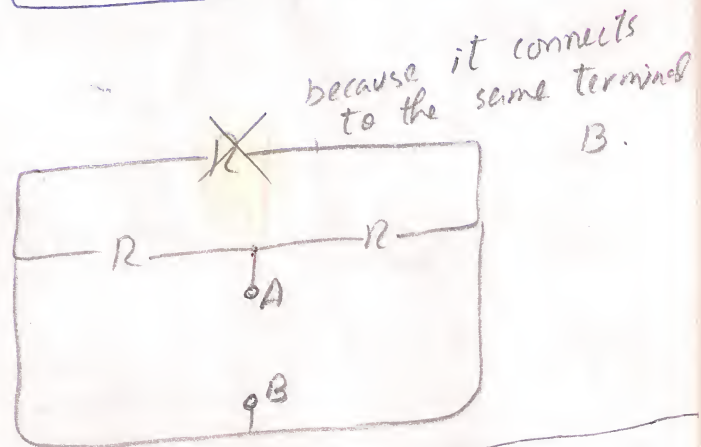
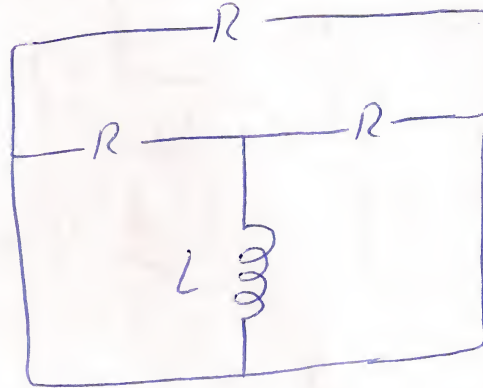
$$= 8.30 \times 0.1 \times 10^{-6} \times 10^3$$

$$\tau = 0.83 ms$$

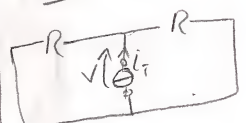
9.3 $\tau = ?$

$$R = 1\Omega$$

$$L = \frac{1}{2} H$$



OR



$$v = \frac{i_T}{\frac{2}{R}}$$

$$v = \frac{1}{2} i_T$$

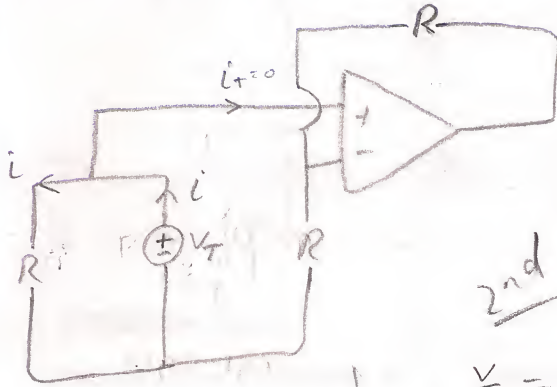
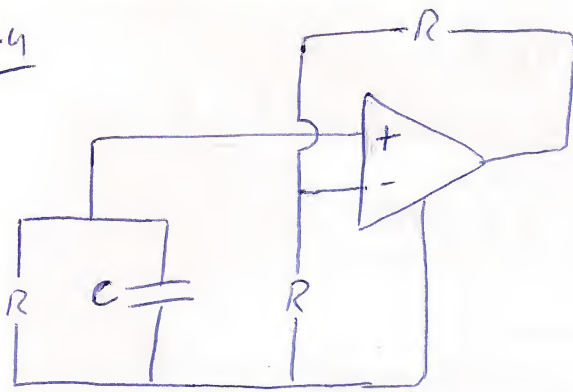
$$R_{eq} = \frac{1}{2}$$

$$R_{eq} = \frac{1}{1+1} = \frac{1}{2}$$

$$\tau = \frac{L}{R_{eq}}$$

$$\tau = 1 s$$

9.4



$$R_{eq} = \frac{V_T}{i}$$

$$\frac{V_T}{i} = R$$

so, $R_{eq} = R$

$$\tau = R.C$$

2nd

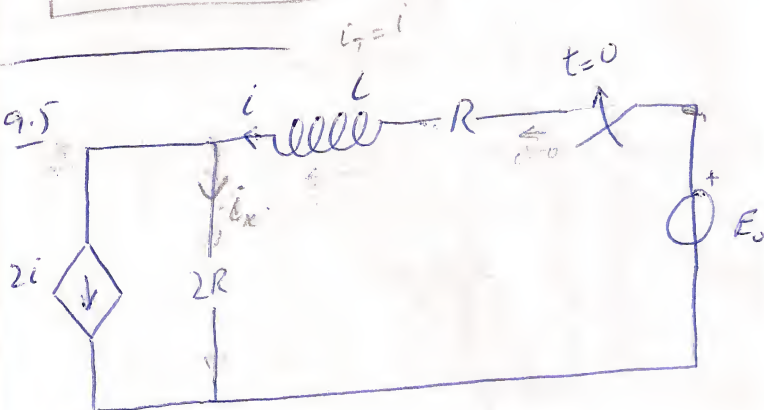
$$\frac{V}{R} = i_T$$

$$V = R i_T + 0$$

$$R_{eq} = R$$

$$\tau = R.C$$

9.5

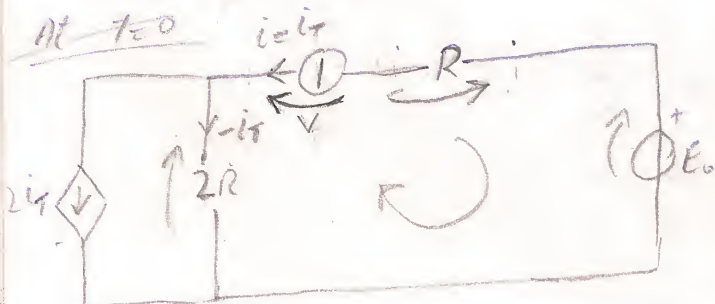


At $t=0$

$$i_x + 2i - i = 0$$

$$i_x = -i$$

At $t=0$

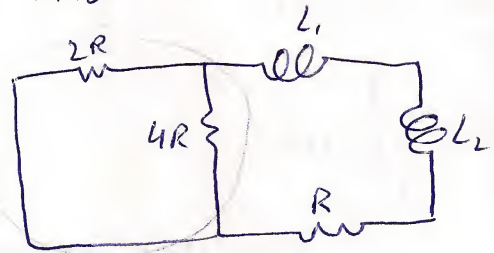


$$i_T R - 2R i_T - E_0 - V = 0$$

$$V = \underbrace{-R i_T}_{R_{eq}} \underbrace{-E_0}_{V_{eq}}$$

$$\tau = \frac{L}{R}$$

9.6 Find τ



$$\frac{8R^2}{6R} = \frac{4}{3}R$$

$$R_{eq} = \frac{4}{3}R + R$$

$$L_{eq} = L_1 + L_2$$

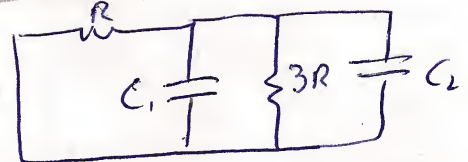
$$R_{eq} = \frac{7}{3}R = \frac{14}{3}$$

$$L_{eq} = 7H$$

$$\tau = \frac{7}{14/3} = \frac{21}{14}$$

$$\tau = 1.5s \quad Ans$$

9.7



$$R_{eq} = \frac{3R^2}{4R} = \frac{3}{4}R$$

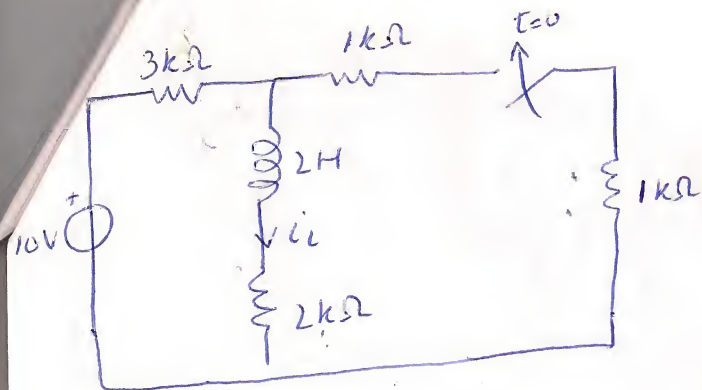
$$C_{eq} = C_1 + C_2$$

$$R_{eq} = \frac{3}{2}k\Omega$$

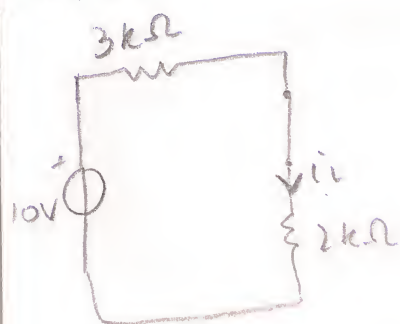
$$C_{eq} = 10\mu F$$

$$\tau = \frac{3}{2} \times 10^{-5} k.\mu s$$

$$\tau = 15ms \quad Ans$$



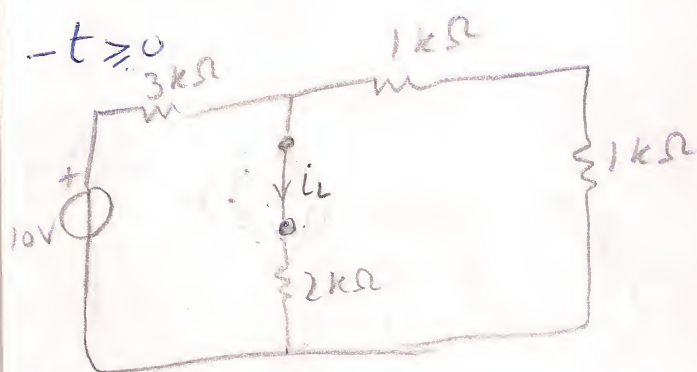
$-t < 0$



$$i_L(0) = \frac{10}{5 \times 10^3}$$

$$i_L(0) = 2 \text{ mA} \quad \checkmark$$

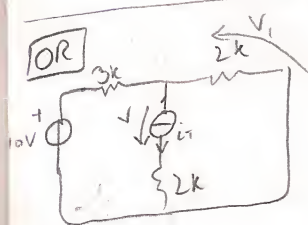
$-t \geq 0$



$$R_{eq} = \frac{6}{5} + 2 = \frac{16}{5} = 3.2 \text{ k}\Omega$$

$$\tau = \frac{L}{R_{eq}} = \frac{2}{3.2 \text{ k}}$$

$$\tau = 0.625 \text{ ms} \quad \text{Ans}$$



$$V_L = \frac{-\frac{10}{3k} + i_s}{\frac{1}{3k} + \frac{1}{2k}} = \frac{-\frac{10}{3k} + \frac{2k}{2k}}{\frac{5}{2.6k}}$$

$$V_L = (-10 + 3k i_s) \frac{2}{5}$$

$$V_L = -4 + \frac{6k}{5} i_s$$

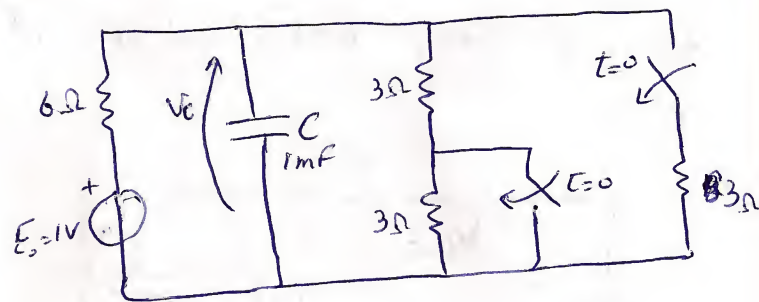
$$V = -V_L + 2k i_s$$

$$V = -4 + \frac{6k}{5} i_s + 2k i_s$$

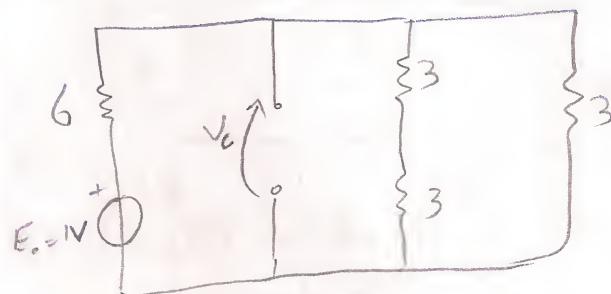
$$V = -4 + \frac{16k}{5} i_s$$

$$R_{eq} = 3.2 \text{ k}$$

4.9 $\tau = ?$, $V_C(0) = ?$



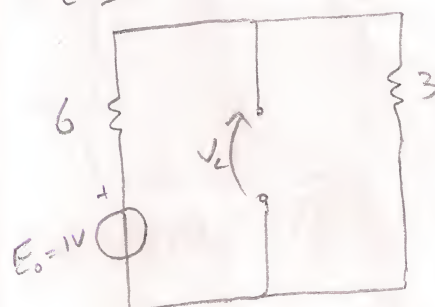
$-t < 0$



$$V_C(0) = \frac{\frac{1}{6}}{\frac{1}{6} + \frac{1}{6} + \frac{1}{3}} = \frac{\frac{1}{6}}{\frac{1+1+2}{6}} = \frac{1}{4}$$

$$V_C(0) = 0.25 \text{ V}$$

$-t \geq 0$



$$R_{eq} = \frac{18}{9} = 2 \Omega$$

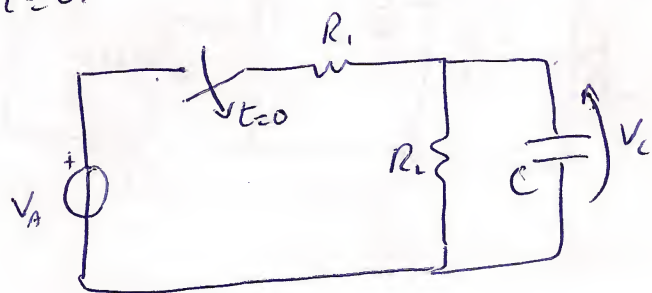
$$\tau = R_{eq} C$$

$$\tau = 2 \text{ ms}$$

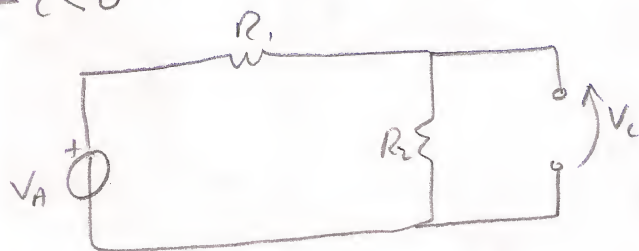
$$\tau = 2 \text{ ms}$$

a.10 9.10

ii) If switch remains closed for a "long time" and open at $T=0$.



$-t < 0$



a) $V_C(0) = \left(\frac{R_2}{R_1 + R_2} \right) V_A$

$-t \rightarrow \infty$



b) $V_C(\infty) = 0V$

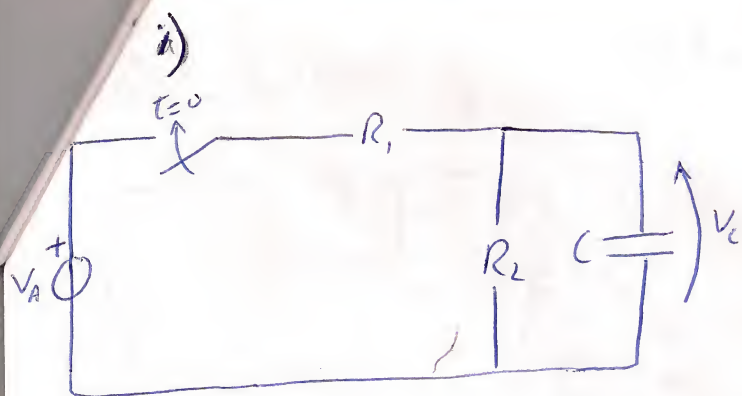
c) $\tau = R_{eq}C$

$R_{eq} = R_2$

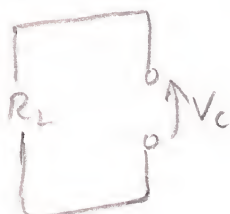
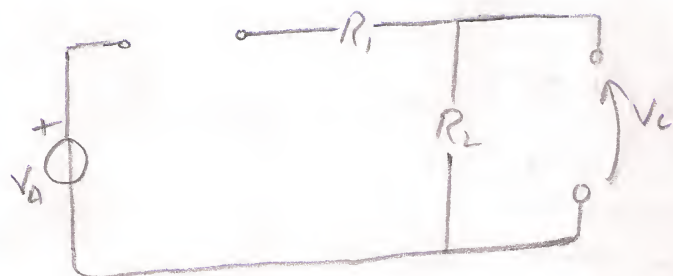
$\tau = R_2C$

d) $V_C(t) = (V_C(0) - V_C(\infty)) e^{\frac{t-t_0}{\tau}} + V_C(\infty)$

$V_C(t) = \left(\frac{R_2 V_A}{R_1 + R_2} \right) e^{-\frac{t}{\tau}}$

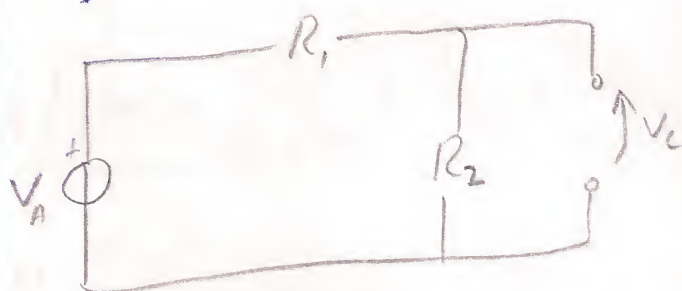


$-t < 0$

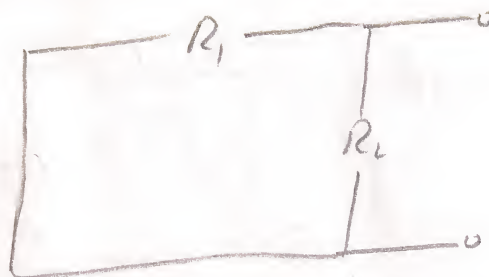


$$V_C(0) = 0$$

$-t \geq 0$



$$V_C = \frac{R_2}{R_1 + R_2} V_A$$



$$R_{eq} = R_1 \parallel R_2$$

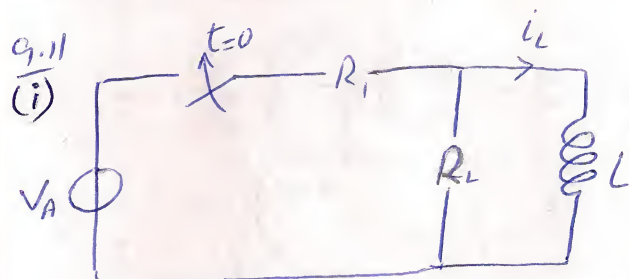
$$\tau = (R_1 \parallel R_2) C$$

As

$$V_C(t) = [V_C(0) - V_{C\infty}] e^{-\frac{t-t_0}{\tau}} + V_{C\infty}$$

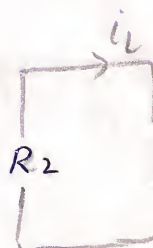
$$= \left(\frac{R_2 V_A}{R_1 + R_2} \right) e^{-\frac{t}{\tau}} + \frac{R_2 V_A}{R_1 + R_2}$$

$$V_C(t) = \frac{R_2 V_A}{R_1 + R_2} (1 - e^{-\frac{t}{\tau}})$$



$V_A = \text{constant}$ (DC steady state)

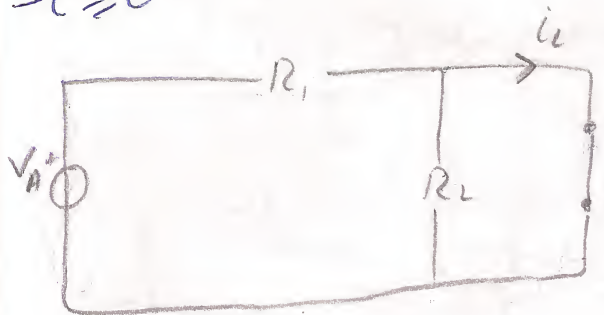
$-t < 0$



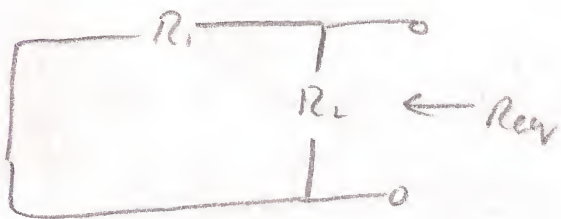
2)

$$i_L(0) = 0 \text{ A}$$

$$-t \geq 0$$



$$b) \quad i_{L\infty} = \frac{V_A}{R_1}$$



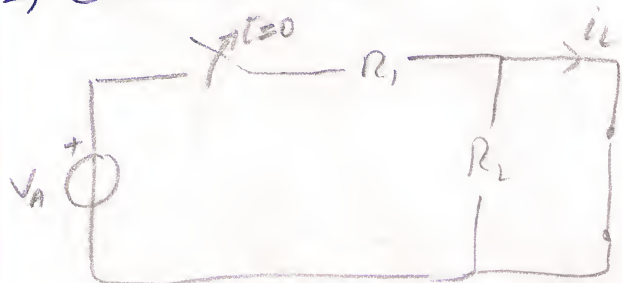
$$R_{eq} = R_1 \parallel R_2$$

$$\tau = \frac{L}{R_1 \parallel R_2}$$

$$i_L(t) = \left(0 - \frac{V_A}{R_1}\right) e^{-t/\tau} + \frac{V_A}{R_1}$$

$$i_L(t) = \frac{V_A}{R_1} (1 - e^{-t/\tau})$$

2) closed-to-open



$$-t < 0$$

$$i_L(0^-) = \frac{V_A}{R_1}$$

$$-t \geq 0$$



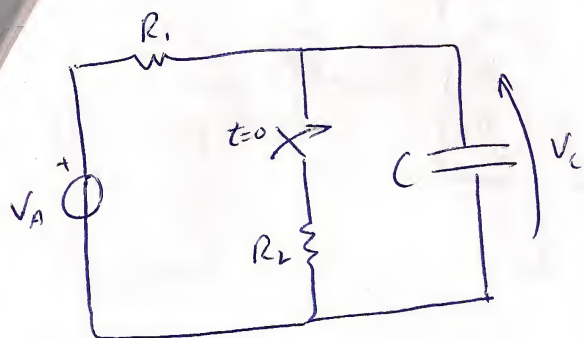
$$i_{L\infty} = 0$$

$$R_{eq} = R_2$$

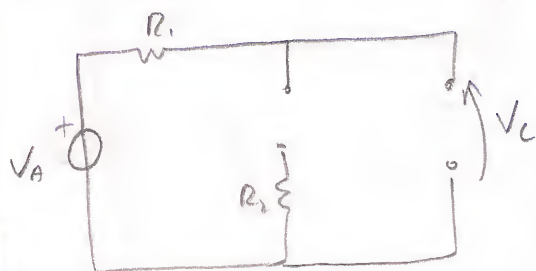
$$\tau = \frac{L}{R_2}$$

$$i_L(t) = \frac{V_A}{R_1} e^{-t/\tau}$$

Open-to-closed

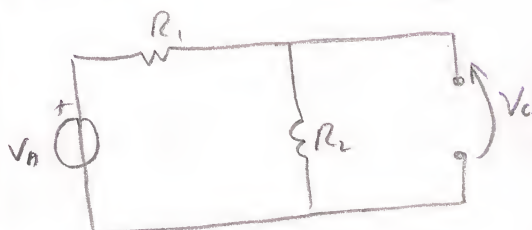


$-t < 0$



a) $V_C(0) = V_A$

$-t \rightarrow \infty$



b) $V_C(\infty) = \frac{R_2 V_A}{R_1 + R_2}$

c) $\tau = ?$

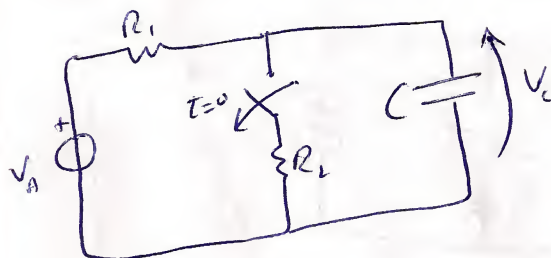
$R_{eq} = R_1 \parallel R_2$

$\tau = (R_1 \parallel R_2) C$

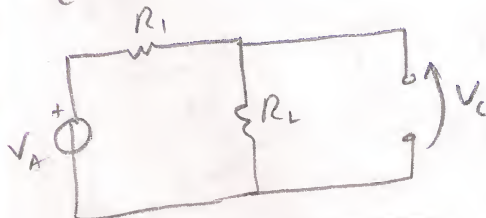
d) $V_C(t) = (V_C(0) - V_C(\infty)) e^{-\frac{t}{\tau}} + V_C(\infty)$
 $= \left(V_A - \frac{V_A R_2}{R_1 + R_2} \right) e^{-\frac{t}{\tau}} + \frac{V_A R_2}{R_1 + R_2}$

$V_C(t) = \frac{V_A R_2}{R_1 + R_2} (1 - e^{-\frac{t}{\tau}}) + V_A e^{-\frac{t}{\tau}}$

ii) Closed-to-open

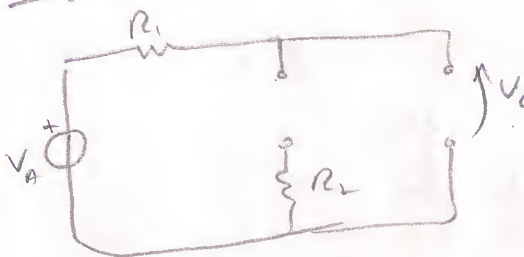


$-t < 0$



a) $V_C(0) = \frac{R_2 V_A}{R_1 + R_2}$

$-t \rightarrow \infty$



b) $V_C(\infty) = V_A$

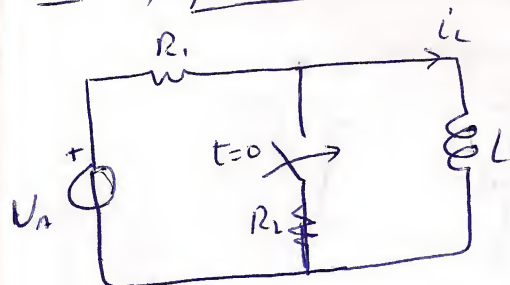
$R_{eq} = R_1$

c) $\tau = R_1 \cdot C$

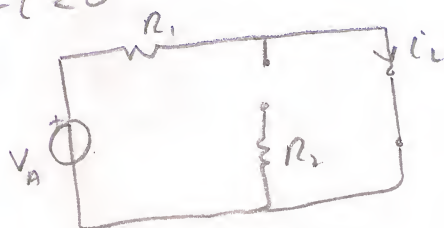
d) $V_C(t) = \left(\frac{V_A R_2}{R_1 + R_2} - V_A \right) e^{-\frac{t}{\tau}} + V_A$

$V_C(t) = \frac{V_A R_2}{R_1 + R_2} e^{-\frac{t}{\tau}} + V_A (1 - e^{-\frac{t}{\tau}})$

9.13 i) Open-to-closed

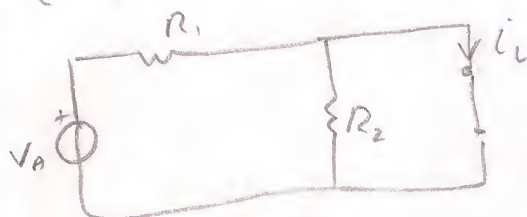


$-t < 0$



a)
$$i_L(0) = \frac{V_A}{R_1}$$

$-t \rightarrow \infty$



b)
$$i_L(\infty) = \frac{V_A}{R_1}$$

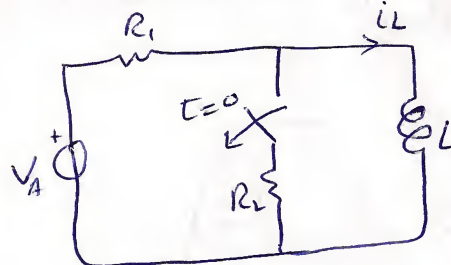
$$R_{eq} = R_1 \parallel R_2$$

c)
$$\tau = \frac{L}{R_1 \parallel R_2}$$

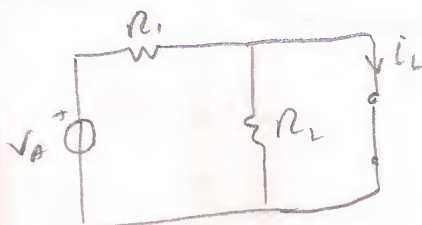
d)
$$i_L(t) = \left(\frac{V_A}{R_1} - \frac{V_A}{R_1} \right) e^{-t/\tau} + \frac{V_A}{R_1}$$

$$i_L(t) = \frac{V_A}{R_1}$$

ii) Closed-to-open

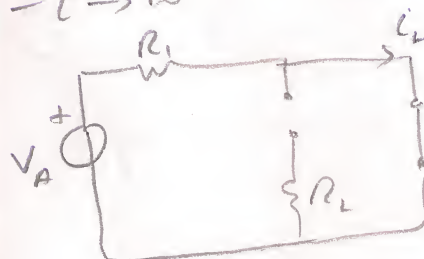


$-t < 0$



a)
$$i_L(0) = \frac{V_A}{R_1}$$

$-t \rightarrow \infty$



b)
$$i_L(\infty) = \frac{V_A}{R_1}$$

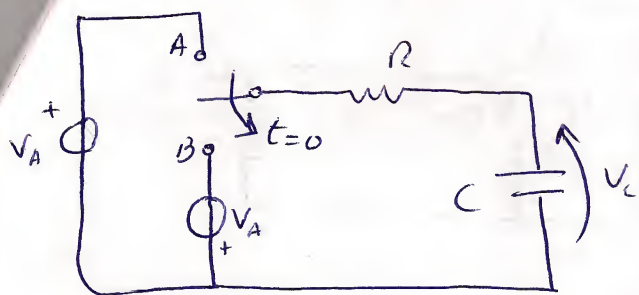
$$R_{eq} = R_1$$

c)
$$\tau = \frac{L}{R_1}$$

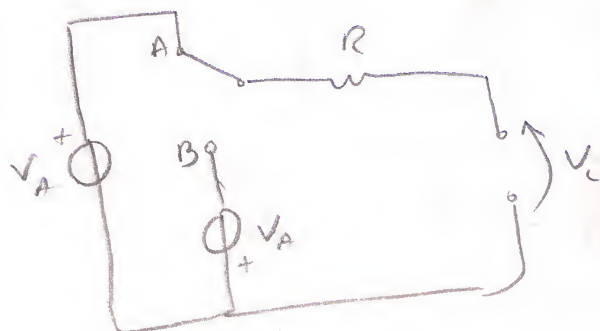
d)
$$i_L(t) = \left(\frac{V_A}{R_1} - \frac{V_A}{R_1} \right) e^{-t/\tau} + \frac{V_A}{R_1}$$

$$i_L(t) = \frac{V_A}{R_1}$$

i) $A \rightarrow B$

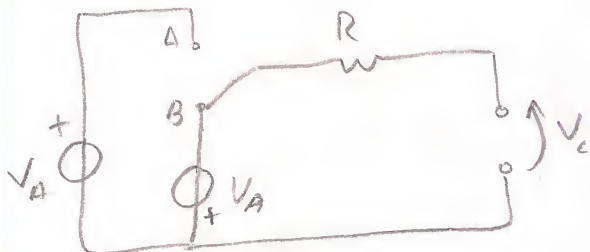


$-t < 0$



$$V_C(0) = V_A$$

$-t \geq 0$



$$V_C(\infty) = -V_A$$

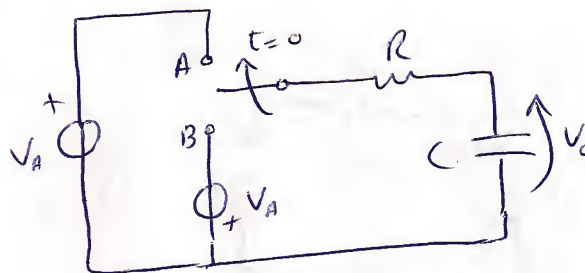
$$R_{eq} = R$$

$$\tau = R \cdot C$$

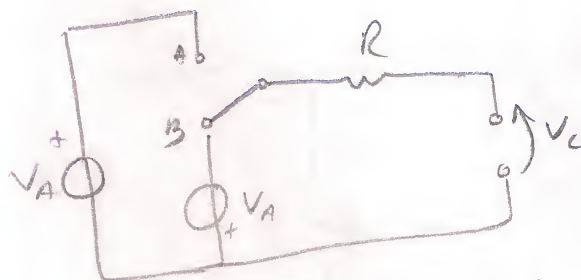
$$V_C(t) = (V_A + V_A) e^{-t/\tau} + V_A$$

$$V_C(t) = -V_A + 2V_A e^{-t/\tau}$$

ii) $B \rightarrow A$

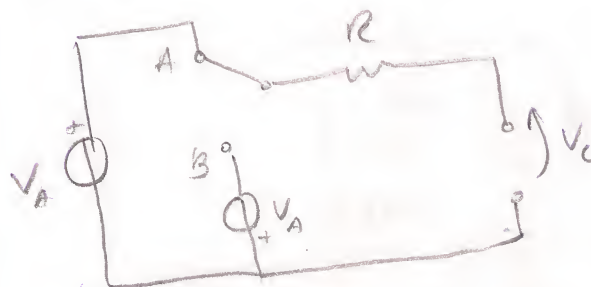


$-t < 0$



$$V_C(0) = -V_A$$

$-t \geq 0$



$$V_C(\infty) = V_A$$

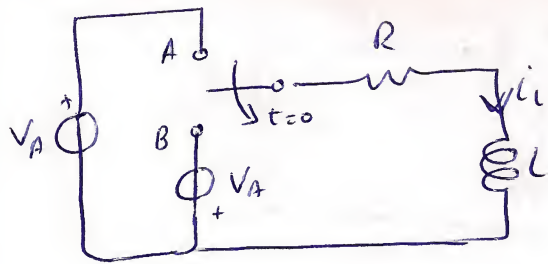
$$R_{eq} = R$$

$$\tau = R \cdot C$$

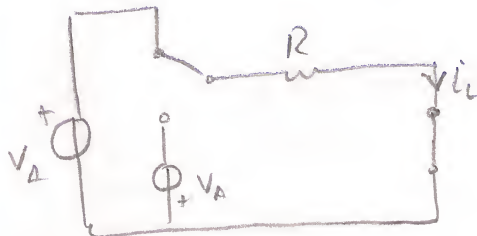
$$V_C(t) = (-V_A + V_A) e^{-t/\tau} + V_A$$

$$V_C(t) = V_A - 2V_A e^{-t/\tau}$$

9.15 i) $A \rightarrow B$

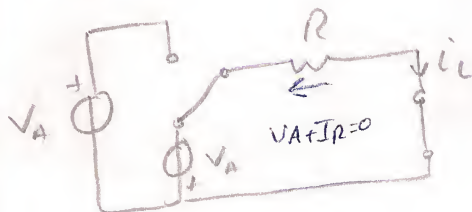


$-t < 0$



$$i_L(0) = +\frac{V_A}{R}$$

$-t \geq 0$



$$i_L(\infty) = -\frac{V_A}{R}$$

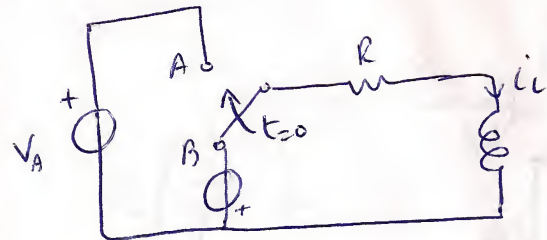
$$R_{eq} = R$$

$$\tau = \frac{L}{R}$$

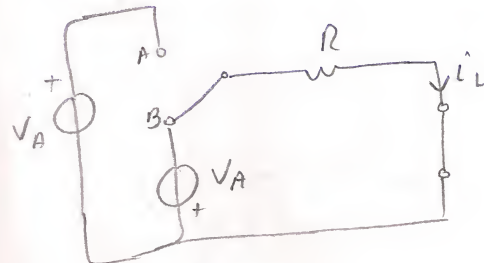
$$i_L(t) = \left(+\frac{V_A}{R} + \frac{V_A}{R} \right) e^{-t/\tau} - \frac{V_A}{R}$$

$$i_L(t) = -\frac{V_A}{R} + \frac{2V_A}{R} e^{-t/\tau}$$

ii) $B \rightarrow A$

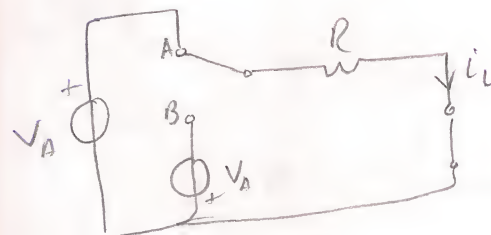


$-t < 0$



$$i_L(0) = -\frac{V_A}{R}$$

$-t \geq 0$



$$i_L(\infty) = +\frac{V_A}{R}$$

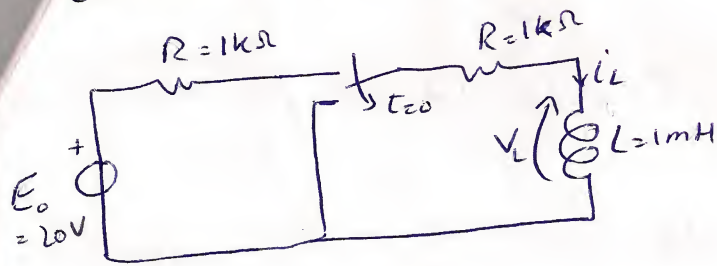
$$R_{eq} = R$$

$$\tau = R \cdot C$$

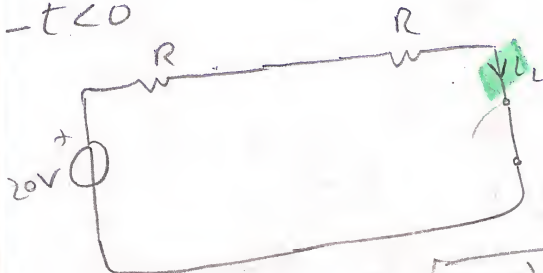
$$i_L(t) = \left(-\frac{V_A}{R} + \frac{V_A}{R} \right) e^{-t/\tau} + \frac{V_A}{R}$$

$$i_L(t) = +\frac{V_A}{R} - \frac{2V_A}{R} e^{-t/\tau}$$

$v_L(0^+)$ & steady state condition $v_L(\infty)$

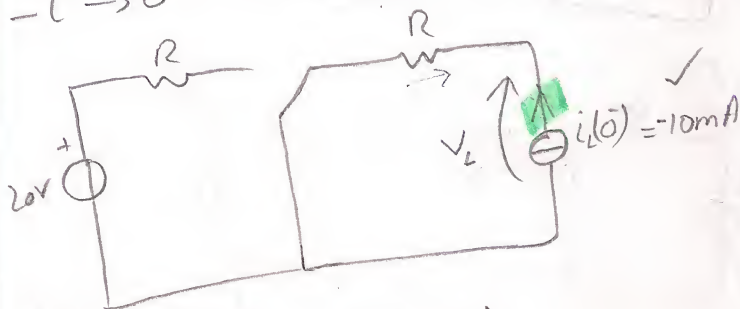


$-t < 0$



$$i_L(0^-) = \frac{20}{2k} \Rightarrow i_L(0^-) = 10 \text{ mA}$$

$-t \rightarrow 0^+$



$$v_L(0^+) = -10 \text{ mA} (1 \text{ k}\Omega)$$

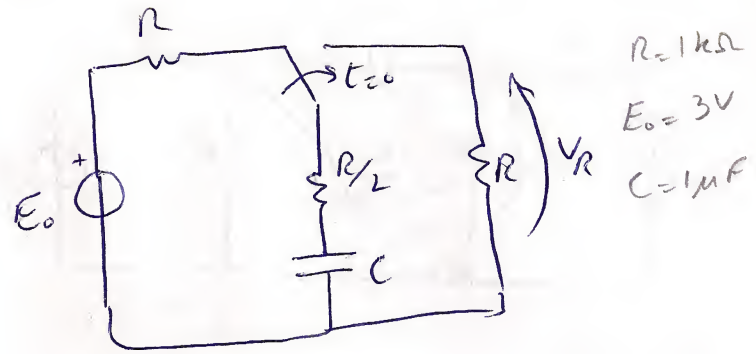
$$v_L(0^+) = -10 \text{ V}$$

$-t \rightarrow +\infty$



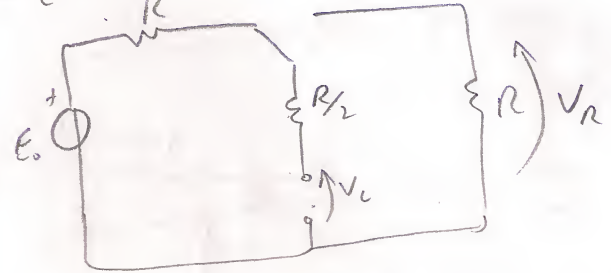
$$v_L(\infty) = 0$$

9.17 $v_R(0^+) = ?$, $v_R(\infty) = ?$



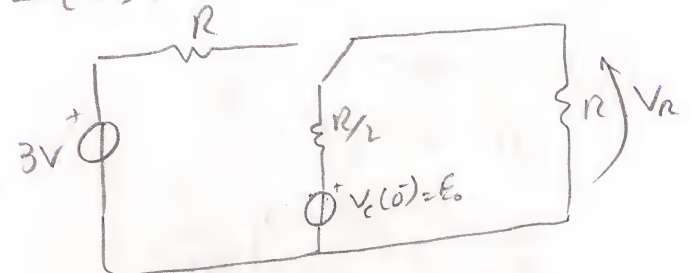
$R = 1 \text{ k}\Omega$
 $E_0 = 3 \text{ V}$
 $C = 1 \mu\text{F}$

$-t < 0$



$$v_R(0^-) = E_0$$

$-t \rightarrow 0^+$



$$v_R(0^+) = \frac{R}{\frac{3R}{2}} \cdot 3$$

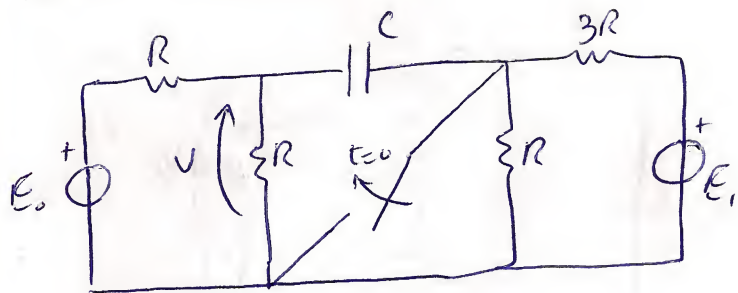
$$v_R(0^+) = 2 \text{ V}$$

$-t \rightarrow \infty$

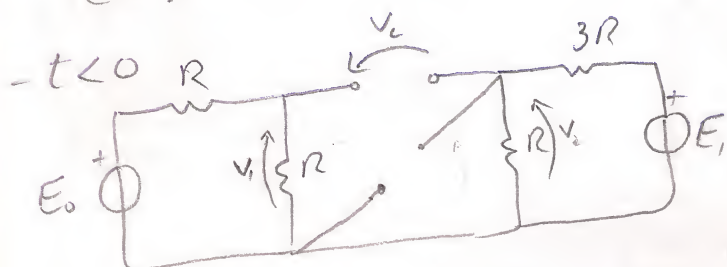


$$v_R(\infty) = 0$$

9.18 $v(t)$ $t > 0$



$E_0 = 12V$
 $E_1 = 12V$
 $R = 4k\Omega$
 $C = 1\mu F$



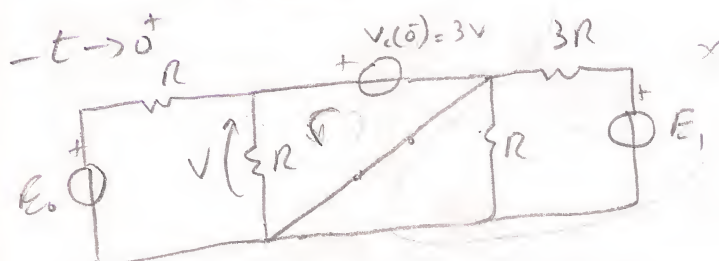
$v_1 = \frac{R}{2R} E_0$, $v_2 = \frac{R}{4R} E_1$

$v_c(0^-) = v_1 + v_2 = 0$

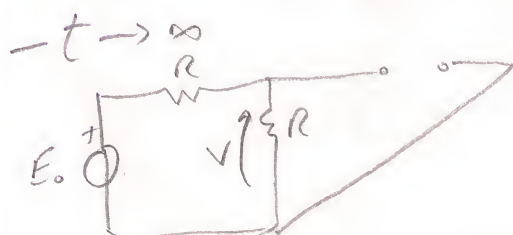
$v_c(0^-) = \frac{1}{2} E_0 - \frac{E_1}{4}$

$v_c(0^-) = 6 - 3$

$v_c(0^-) = 3V$



$v(0^+) = v_c(0^-) \Rightarrow v(0^+) = 3V$



$v(\infty) = \frac{E_0}{2} \Rightarrow v(\infty) = 6V$

$R_{eq} = \frac{R}{2} = 2k\Omega$

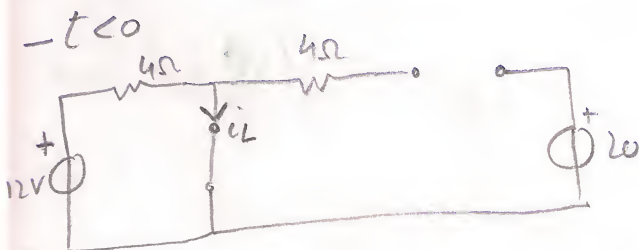
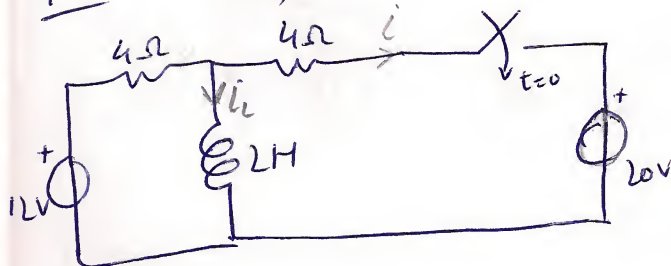
$\tau = R_{eq} C$

$\tau = 2k \cdot 1\mu \Rightarrow \tau = 2ms$

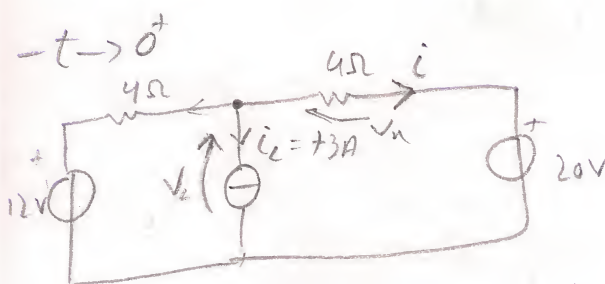
$v(t) = (3 - 6)e^{-t/\tau} + 6$

$v(t) = -3e^{-t/\tau} + 6$

9.19 $i(t)$ $t > 0$



$i_L(0^-) = +\frac{12}{4} \Rightarrow i_L(0^-) = +3A$

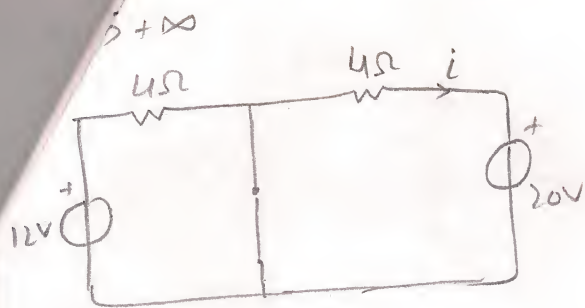


$v_L = \frac{3 + 5 \cdot 3}{1/2} \Rightarrow v_L = 10V$

$v_x - v_L + 20 = 0$

$v_x = -10V$

$i(0^+) = -2.5A$



$$i(\infty) = -\frac{20}{4}$$

$$\boxed{i(\infty) = -5}$$

$$i(t) = (-2.5 + 5)e^{-\frac{t}{\tau}} - 5$$

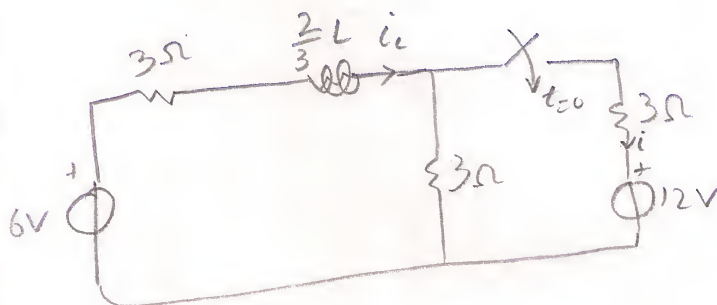
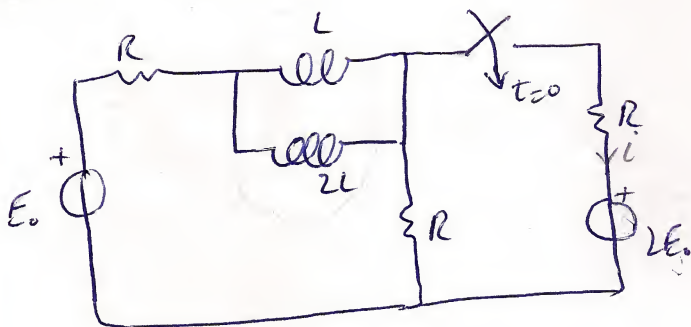
$$\boxed{i(t) = 2.5e^{-\frac{t}{\tau}} - 5}$$

$$R_{eq} = \frac{16^2}{8} = 2\Omega$$

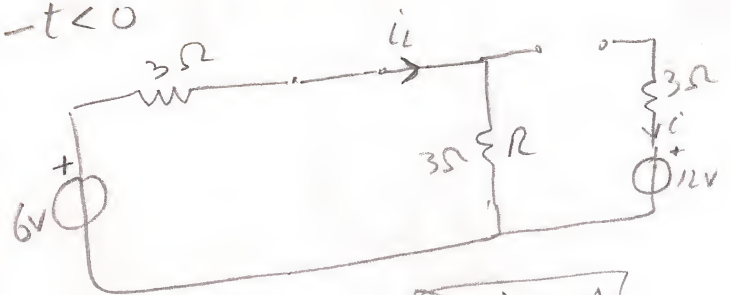
$$\tau = \frac{L}{R_{eq}}$$

$$\tau = \frac{2}{2} \Rightarrow \boxed{\tau = 1s}$$

9.20 $i(t) = ? \quad t > 0$

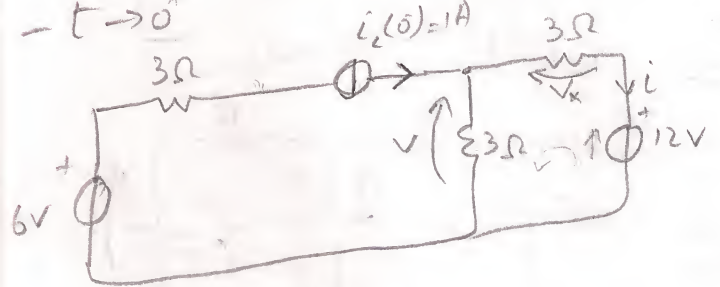


$$-t < 0$$



$$i_L(0^-) = \frac{6}{6} \Rightarrow \boxed{i_L(0^-) = 1A}$$

$$-t \rightarrow 0^+$$



$$V = \frac{1+4}{\frac{1}{3} + \frac{1}{3}} = \frac{5}{\frac{2}{3}} = \frac{15}{2}$$

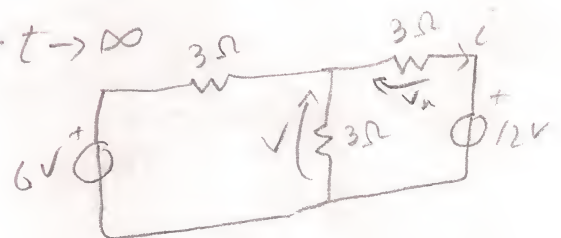
$$V = 7.5V$$

$$12 + V_x - 7.5 = 0 \Rightarrow V_x = -4.5V$$

$$i(0^+) = -\frac{4.5}{3}$$

$$\boxed{i(0^+) = -1.5A}$$

$$-t \rightarrow \infty$$



$$V = \frac{6 + \frac{12}{3}}{\frac{1}{3} + \frac{1}{3} + \frac{1}{3}} = \frac{2+4}{1}$$

$$V = 6V$$

$$V_x + 12 - 6 = 0 \Rightarrow V_x = -6V$$

$$i(\infty) = -\frac{6}{3} \Rightarrow \boxed{i(\infty) = -2A}$$

$$i(t) = (-1.5 + 2)e^{-\frac{t}{\tau}} - 2$$

$$\boxed{i(t) = 0.5e^{-\frac{t}{\tau}} - 2}$$

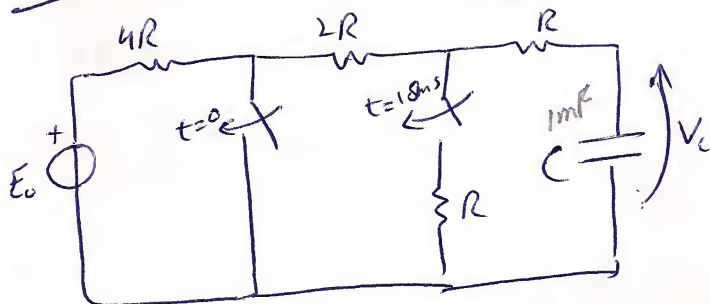
$$R_{eq} = \frac{9}{6} + 3 = \frac{27}{6} = \frac{9}{2}$$

$$R_{eq} = \frac{9}{2}$$

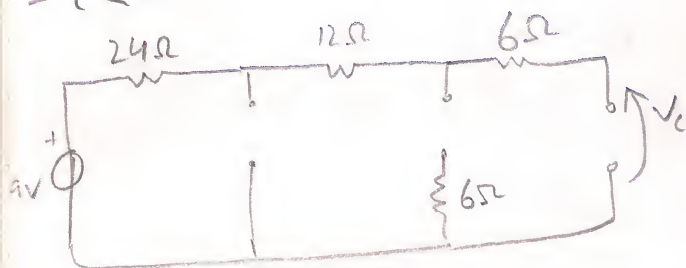
$$\tau = \frac{2.7}{\frac{9}{2}} = \frac{4}{9}$$

$$\tau = 0.445$$

$$9.21 \quad v_c(t) \quad t > 0$$

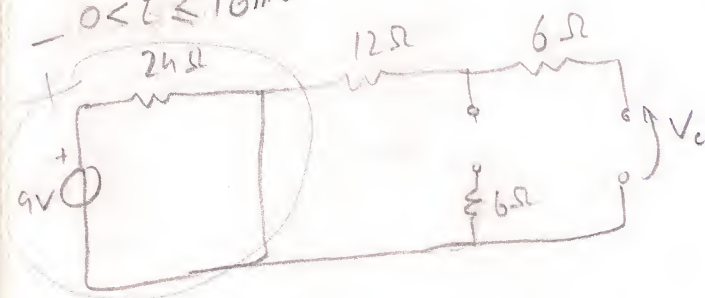


$$-t < 0$$



$$v_c(0) = 9V$$

$$-0 < t \leq 18ms$$



$$v_c = 0V$$

$$v_c(t) = 9e^{-t/\tau'} V$$

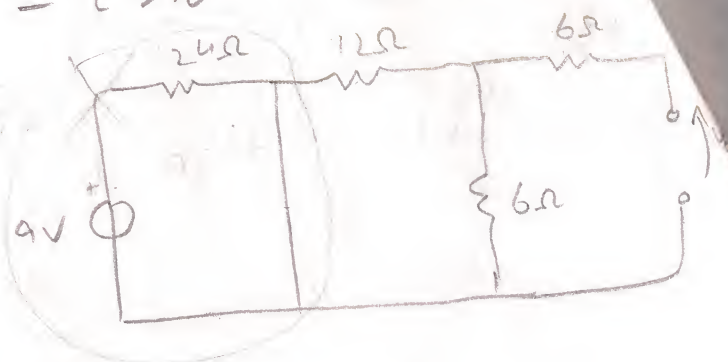
$$R_{eq} = 18\Omega$$

$$\tau' = 18ms$$

$$v_c(18) = 9e^{-1}$$

$$v_c(18) = 3.31 V$$

$$-t > 18$$



$$v_c(\infty) = 0$$

$$v_c(t) = 3.31 e^{-\frac{(t-18)}{\tau''}}$$

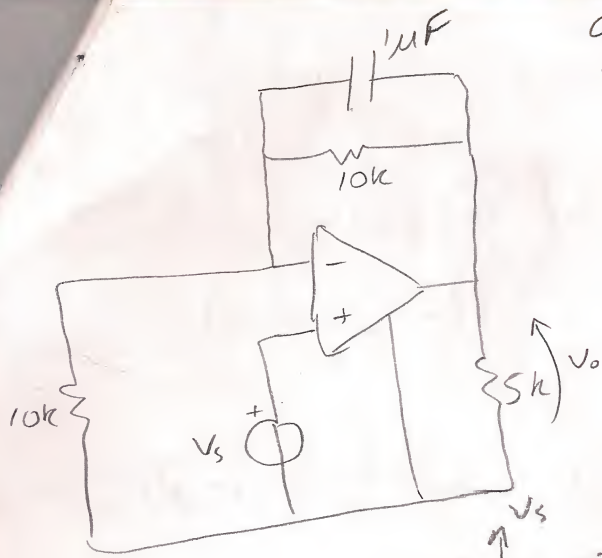
$$R_{eq} = \frac{12 \cdot 6}{18} + 6$$

$$R_{eq} = 10\Omega \quad \checkmark$$

$$\tau'' = 10ms$$

9.22

$-T < t < 2T$

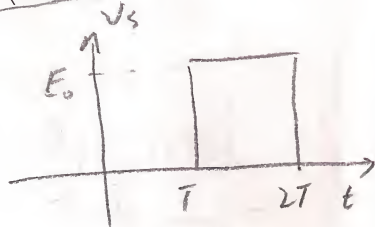


$-t < T$

$$V_s = 0$$

$$V_c(t) = 0$$

$$V_o(t) = 0$$



$$\frac{V_c}{10k} = -\frac{10}{10k}$$

$$V_c = -10V$$

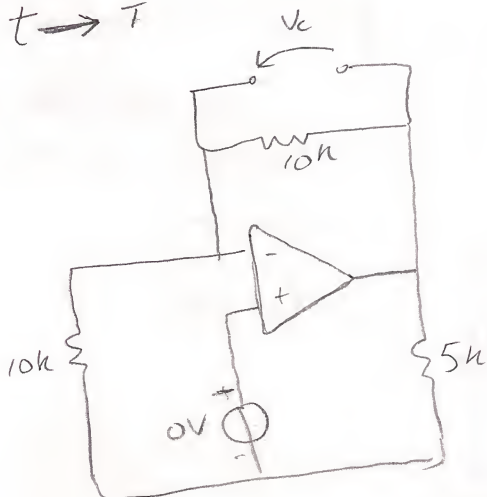
$T < t < 2T$

$$V_o + V_c - 10 = 0$$

$$V_o = 20V$$

$T < t < 2T$

$-t \rightarrow T$



$$V_c(t) = 10e^{-\frac{t-T}{\tau}} - 10V$$

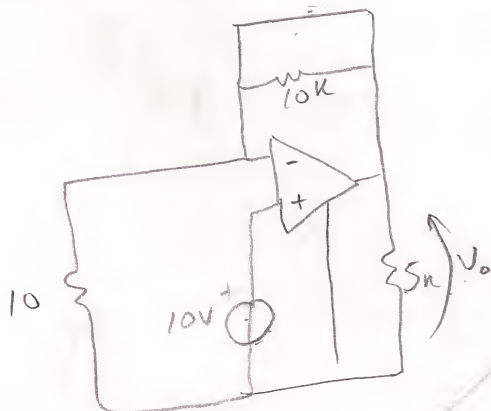
$T < t < 2T$

$$V_o(t) = (10 - 20)e^{-\frac{t-T}{\tau}} + 20V$$

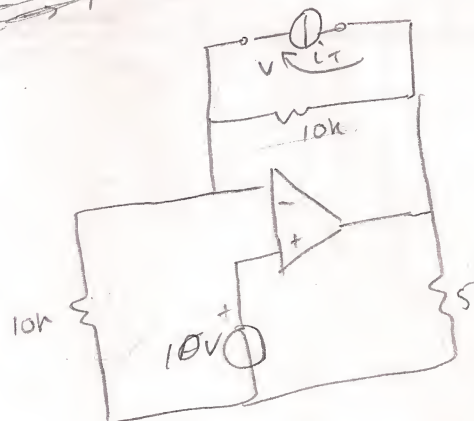
For τ $T < t < 2T$

$$V_c(T^-) = 0, \quad V_c(T^+) = V_c(T^-) = 0V$$

$-t \rightarrow T^+$



$$V_o(T^+) = 10V$$



$$\frac{V}{10k} + \frac{10}{10k} = i_T$$

$$\frac{V}{10k} = i_T - \frac{10}{10k}$$

$$V = 10k i_T - 10$$

$$R_{eq} = 10k$$

$$\tau = 10ms$$

$$-t \rightarrow 2T^+$$

$$V_c(2T^+) = V_c(2T^-) = 10e^{-\frac{T}{\tau}} - 10 = 10e^{-\frac{1}{2}} - 10 = -3.93V$$

$$V_o(2T^+) = -V_c(2T^+) \checkmark$$

$$V_o(2T^+) = 3.93V$$

$$-t > 2T$$

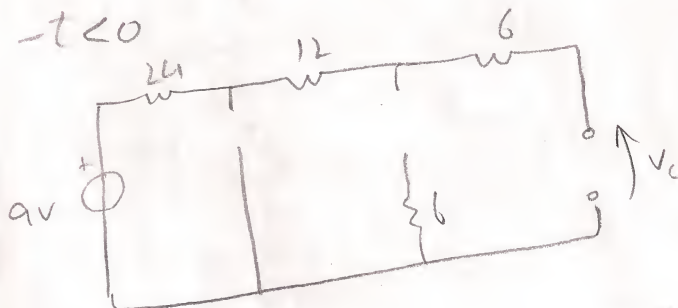
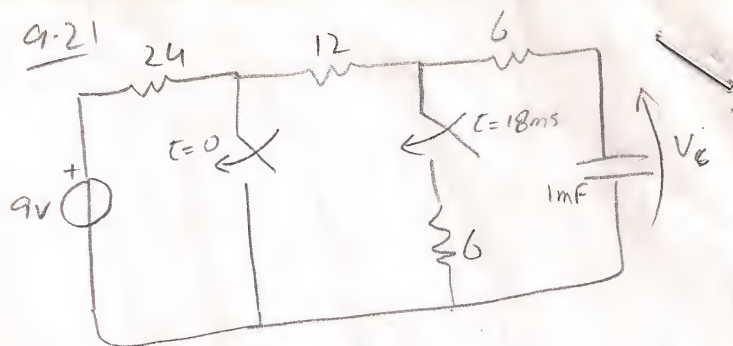
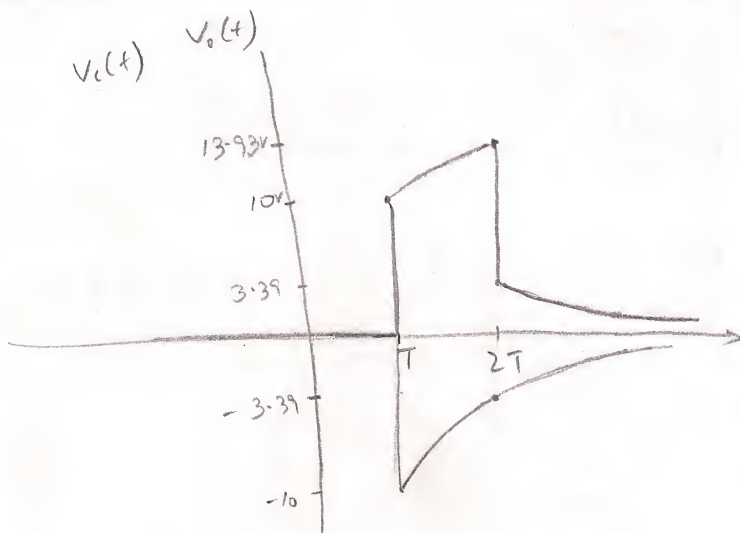
$$V_c(\infty) = 0$$

$$V_o(\infty) = 0 \checkmark$$

$$V_c(t) = -3.93 e^{-\frac{t-2T}{\tau}}$$

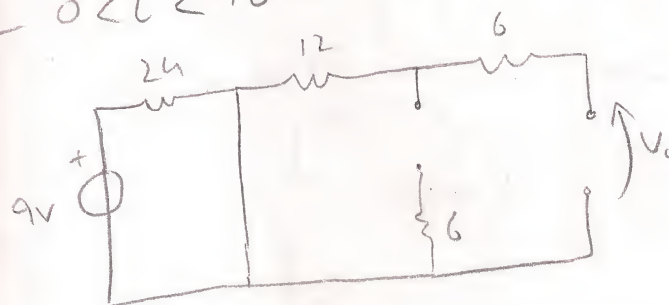
$$V_o(t) = 3.93 e^{-\frac{t-2T}{\tau}}$$

$$t > 2T$$



$$V_c(0^-) = 9V$$

$$-0 < t < 18ms$$



$$V_c = 0 \quad 0 < t < 18ms$$

$$R_{eq} = 18\Omega$$

$$\tau = 18ms$$

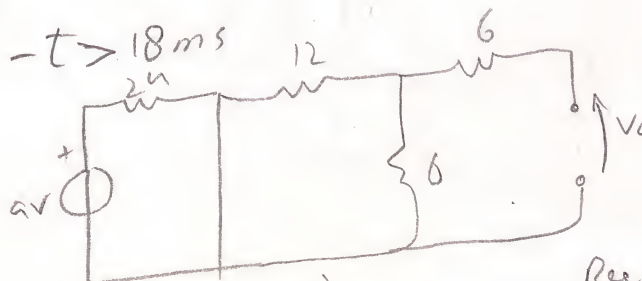
$$V_c(t) = 9 e^{-\frac{t}{\tau}}$$

$$0 < t < 18$$

$$-t \rightarrow 18ms^+$$

$$V_c(18ms^+) = V_c(18ms^-) = 9 e^{-\frac{18ms}{18ms}} = 3.31V$$

$$-t > 18ms$$



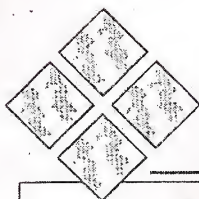
$$V_c(\infty) = 0$$

$$R_{eq} = 10\Omega$$

$$\tau' = 10.1ms$$

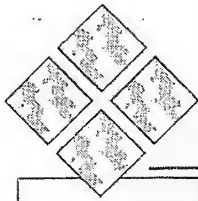
$$\tau'' = 10ms$$

CH#10



BASIC LAPLACE TRANSFORM PAIRS

<u>SIGNAL</u>	<u>WAVEFORM $f(t)$</u>	<u>TRANSFORM $F(s)$</u>
Impulse	$\delta(t)$	1
Step Function	$u(t)$ $\mathcal{L}u(\beta t + E)$	$\frac{1}{s} \cdot \frac{\alpha}{s} e^{\frac{E}{\beta}}$
Ramp	$tu(t)$	$\frac{1}{s^2}$
Exponential	$[e^{-\alpha t}]u(t)$	$\frac{1}{s + \alpha}$
Damped Ramp	$[te^{-\alpha t}]u(t)$	$\frac{1}{(s + \alpha)^2}$
Sine	$[\sin \beta t]u(t)$	$\frac{\beta}{s^2 + \beta^2}$
Cosine	$[\cos \beta t]u(t)$	$\frac{s}{s^2 + \beta^2}$
Damped Sine	$[e^{-\alpha t} \sin \beta t]u(t)$	$\frac{\beta}{(s + \alpha)^2 + \beta^2}$
Damped Cosine	$[e^{-\alpha t} \cos \beta t]u(t)$	$\frac{(s + \alpha)}{(s + \alpha)^2 + \beta^2}$
Simple Complex Poles	$[2 k e^{-\alpha t} \cos(\beta t + \angle k)]u(t)$	$\frac{k}{s + \alpha - j\beta} + \frac{k^*}{s + \alpha + j\beta}$
Double Complex Poles	$[2 k te^{-\alpha t} \cos(\beta t + \angle k)]u(t)$	$\frac{k}{(s + \alpha - j\beta)^2} + \frac{k^*}{(s + \alpha + j\beta)^2}$



BASIC LAPLACE TRANSFORMATION PROPERTIES

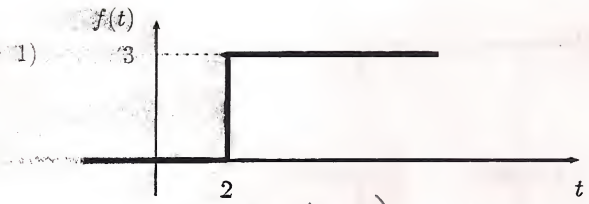
PROPERTIES	TIME DOMAIN	FREQUENCY DOMAIN
Independent Variable	t	s
Signal Representation	$f(t)$	$F(s)$
Uniqueness	$\mathcal{L}^{-1}\{F(s)\} \Leftrightarrow [f(t)]u(t)$	$\mathcal{L}\{f(t)\} = F(s)$
Linearity	$Af_1(t) + Bf_2(t)$	$AF_1(s) + BF_2(s)$
Integration	$\int_0^t f(\tau) d\tau$	$\frac{F(s)}{s}$
Differentiation	$\frac{df(t)}{dt}$ $\frac{d^2 f(t)}{dt^2}$ $\frac{d^3 f(t)}{dt^3}$	$sF(s) - f(0-)$ $s^2 F(s) - sf(0-) - f'(0-)$ $s^3 F(s) - s^2 f(0-) - sf'(0-) - f''(0-)$
t-Translation	$[f(t-a)]u(t-a)$	$e^{-as} F(s)$
s-Translation	$e^{-at} f(t)$	$F(s+a)$
Scaling	$f(at)$	$\frac{1}{a} F\left(\frac{s}{a}\right)$
Final Value	$\lim_{t \rightarrow \infty} f(t)$	$\lim_{s \rightarrow 0} sF(s)$
Initial Value	$\lim_{t \rightarrow 0+} f(t)$	$\lim_{s \rightarrow \infty} sF(s)$



$$\frac{A}{T} u(t) - \frac{A}{T} u(t-T)$$

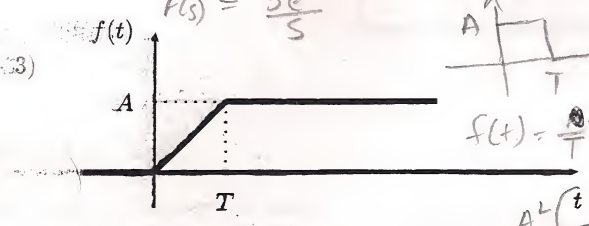
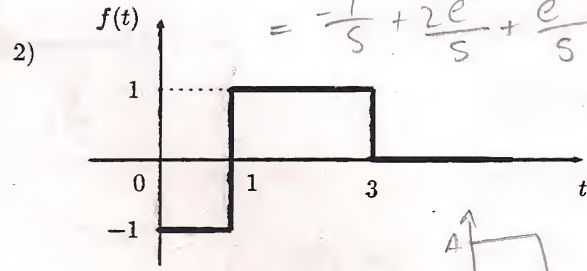
$$-u(t) + 2u(t-1) + u(t-3)$$

$$= -\frac{1}{s} + 2\frac{e^{-s}}{s} + \frac{e^{-3s}}{s}$$



$$f(t) = 3u(t-2)$$

$$F(s) = \frac{3e^{-2s}}{s}$$



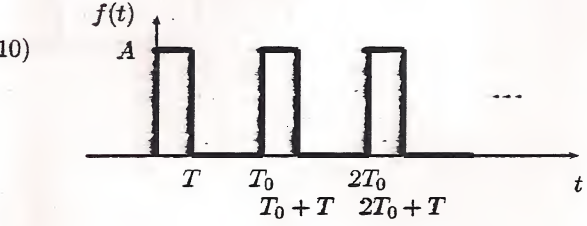
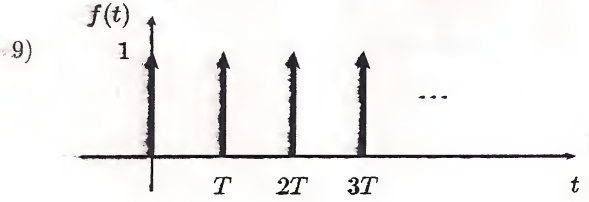
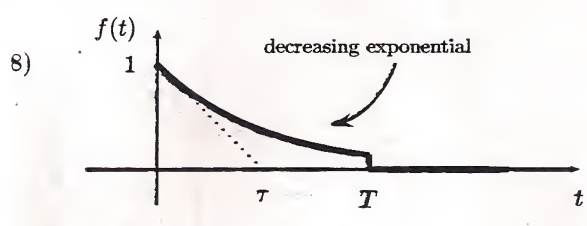
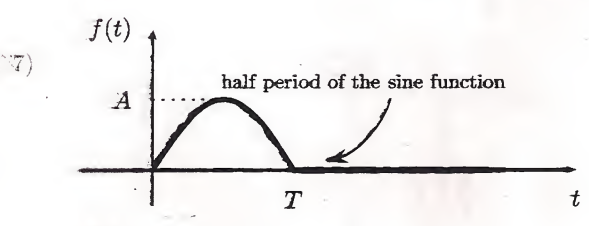
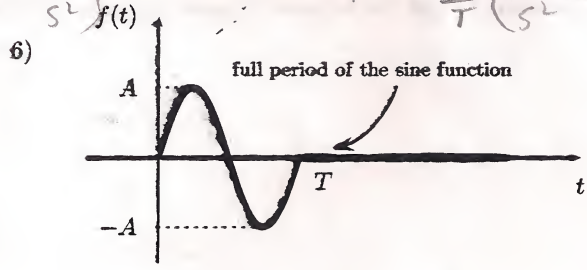
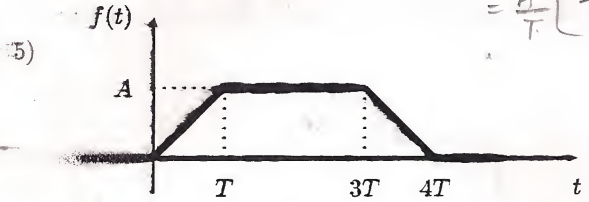
4)

$$f(t) = \frac{A}{T} [Au(t) - Au(t-T)]$$

$$= \frac{A^2}{T} \left[\frac{t}{s^2} - \frac{e^{-Ts}}{s^2} \right]$$

$$\frac{A}{T} [Au(t) - 2Au(t-T) + Au(t-2T)]$$

$$\frac{A^3}{T} \left[\frac{t^2}{s^3} - \frac{2Te^{-Ts}}{s^3} + \frac{e^{-2Ts}}{s^3} \right]$$



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- i) $f(t) = 3u(t-2)$
- ii) $f(t) = -u(t) + 2u(t-1) - u(t-3)$
- iii) $f(t) = \frac{A}{T} \delta(t) - \frac{A}{T} \delta(t-T)$
- iv) $f(t) = \frac{A}{T} \delta(t) - \frac{2A}{T} \delta(t-T) + \frac{A}{T} \delta(t-2T)$
- v) $f(t) = \frac{A}{T} \delta(t) - \frac{A}{T} \delta(t-T) + \frac{A}{T} \delta(t-3T) - \frac{A}{T} \delta(t-4T)$

Calculate Laplace transform.

$$1) f(t) = e^{-at} - e^{-bt}$$

$$F(s) = \frac{1}{s+a} - \frac{1}{s+b}$$

$$2) f(t) = \sin(\omega t + \theta)$$

$$F(s) = \mathcal{L}\{f(t)\}$$

$$= \frac{e^{j(\omega t + \theta)} - e^{-j(\omega t + \theta)}}{2j}$$

$$= \frac{e^{j\theta}}{2j} \cdot e^{j\omega t} - \frac{e^{-j\theta}}{2j} \cdot e^{-j\omega t}$$

$$= \frac{e^{j\theta}}{2j} \cdot \frac{1}{s-j\omega} - \frac{e^{-j\theta}}{2j} \cdot \frac{1}{s+j\omega}$$

$$= \frac{e^{j\theta}(s+j\omega) - e^{-j\theta}(s-j\omega)}{2j(s-j\omega)(s+j\omega)}$$

$$= \frac{\left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right)s + \frac{e^{j\theta} + e^{-j\theta}}{2j}j\omega}{s^2 + \omega^2}$$

$$= \frac{\sin\theta s + \cos\theta \omega}{s^2 + \omega^2}$$

$$F(s) = \frac{s \sin\theta + \omega \cos\theta}{s^2 + \omega^2} \quad R$$

$$3) f(t) = e^{-\alpha t} \cos(\omega t + \theta)$$

$$F(s) = \frac{e^{-\alpha t} e^{j(\omega t + \theta)} + e^{-\alpha t} e^{-j(\omega t + \theta)}}{2}$$

$$= \frac{e^{j\theta}}{2} \cdot e^{(-\alpha + j\omega)t} + \frac{e^{-j\theta}}{2} \cdot e^{(-\alpha - j\omega)t}$$

$$= \frac{e^{j\theta}}{2} \cdot \frac{1}{s + \alpha - j\omega} + \frac{e^{-j\theta}}{2} \cdot \frac{1}{s + \alpha + j\omega}$$

$$= \frac{e^{j\theta}(s + \alpha + j\omega) + e^{-j\theta}(s + \alpha - j\omega)}{2(s + \alpha - j\omega)(s + \alpha + j\omega)}$$

$$= \frac{\left(\frac{e^{j\theta} + e^{-j\theta}}{2}\right)(s + \alpha) + \left(\frac{e^{j\theta} - e^{-j\theta}}{2j}\right)j\omega}{(s + \alpha)^2 + \omega^2}$$

$$= \frac{\cos\theta (s + \alpha) + \sin\theta j^2 \omega}{(s + \alpha)^2 + \omega^2}$$

$$= \frac{(s + \alpha) \cos\theta - \omega \sin\theta}{(s + \alpha)^2 + \omega^2}$$

$$4) f(t) = e^{-2t} + \sin t$$

$$F(s) = \frac{1}{s+2} + \frac{1}{s^2+1}$$

$$5) f(t) = e^{-4t} u(t-3)$$

$$f_1(t) = u(t-3) \xrightarrow{\mathcal{L}} F_1(s) = \frac{e^{-3s}}{s}$$

$$f(t) = e^{-4t} u(t-3) \xrightarrow{\mathcal{L}} F(s) = F_1(s+4)$$

$$F(s) = \frac{e^{-3(s+4)}}{s+4}$$

$$\left. \begin{aligned} &\sin\omega t \cos\theta + \cos\omega t \sin\theta \\ &\frac{\cos\theta \omega}{s^2 + \omega^2} + \frac{\sin\theta s}{s^2 + \omega^2} \end{aligned} \right\}$$

$$e^{-4t+12-12} u(t-3)$$

$$e^{-12} e^{-4(t-3)} u(t-3)$$

$$e^{-12-3s} \frac{1}{s+4} \left(\frac{e^{-3(s+4)}}{s+4} \right)$$

$$= e^{-12-3s} \frac{1}{s+4} = e^{-3(s+4)} \frac{1}{s+4}$$

$$6) f(t) = \sin(t - \tau) u(t - \tau)$$

$$= F(s) e^{-\tau s} = \mathcal{L}\{\sin t u(t)\} e^{-\tau s}$$

$$F(s) = \frac{1}{s^2 + 1} e^{-\tau s}$$

$$7) f(t) = \delta(t) + 2u(t) - 3e^{-2t}$$

$$= 1 + \frac{2}{s} - \frac{3}{s+2}$$

$$8) f(t) = \delta(t) - \delta(t - \tau)$$

$$= 1 - \mathcal{L}\{\delta(t) u(t)\} e^{-\tau s}$$

$$= 1 - e^{-\tau s}$$

$$9) f(t) = \sum_{n=0}^{\infty} \delta(t - nT)$$

$$= \sum_{n=0}^{\infty} \mathcal{L}\{\delta(t)\} e^{-nTs}$$

$$= \sum_{n=0}^{\infty} e^{-nTs}$$

$$\text{let } |x| = e^{-Ts}$$

$$= \sum_{n=0}^{\infty} |x|^n$$

$$\text{if } |x| < 1$$

$$= \frac{1}{1 - |x|}$$

$$= \frac{1}{1 - e^{-Ts}} R$$

$$10) f(t) = \sum_{n=0}^{\infty} (-1)^n u(t - nT)$$

$$= \sum_{n=0}^{\infty} (-1)^n \mathcal{L}\{u(t)\} e^{-sTn}$$

$$= \sum_{n=0}^{\infty} (-1)^n \frac{1}{s} e^{-sTn}$$

$$= \sum_{k=0}^{\infty} \frac{1}{s} (e^{-sT})^{2k} - \sum_{k=0}^{\infty} \frac{1}{s} (e^{-sT})^{2k+1}$$

$$= \sum_{k=0}^{\infty} (e^{-sT})^{2k} \left[\frac{1}{s} - \frac{1}{s} e^{-sT} \right]$$

$$= \frac{1}{s} (1 - e^{-sT}) \sum_{k=0}^{\infty} e^{-2skT}$$

$$= \frac{1}{s} (1 - e^{-sT}) \cdot \frac{1}{1 - e^{-2sT}}$$

$$= \frac{1}{s} \cdot \frac{1 - e^{-sT}}{1 - e^{-2sT}} R$$

$$11) f(t) = u(4t)$$

$$= \frac{1}{s}$$

$$12) f(t) = u(4t - 3)$$

$$= u\left[4\left(t - \frac{3}{4}\right)\right]$$

$$= \frac{1}{s} e^{-\frac{3}{4}s} R$$

0.2
Calculate inverse Laplace

$$1) F(s) = \frac{3}{s} - \frac{5}{s+1}$$

$$= 3u(t) - 5e^{-t}u(t)$$

$$f(t) = (3 - 5e^{-t})u(t)$$

$$2) F(s) = \frac{6}{s^2 + 4}$$

$$= 3 \cdot \frac{2}{s^2 + 2^2}$$

$$= 3 \sin(2t) u(t)$$

$$3) F(s) = \frac{s^2 + 12}{s(s+2)(s+3)}$$

$$= \frac{A_1}{s} + \frac{A_2}{s+2} + \frac{A_3}{s+3} \quad \text{PC}$$

$$A_1 = \lim_{s \rightarrow 0} s F(s) = \frac{s^2 + 12}{(s+2)(s+3)} \Big|_0 = 2$$

$$A_2 = \lim_{s \rightarrow -2} (s+2) F(s) = \frac{s^2 + 12}{s(s+3)} \Big|_{-2} = \frac{16}{-2} = -8$$

$$A_3 = \lim_{s \rightarrow -3} (s+3) F(s) = \frac{s^2 + 12}{s(s+2)} \Big|_{-3} = \frac{21}{3} = 7$$

$$F(s) = \frac{2}{s} + \frac{-8}{s+2} + \frac{7}{s+3}$$

$$f(t) = (2 - 8e^{-2t} + 7e^{-3t})u(t) \quad \text{Ans}$$

$$4) F(s) = \frac{6(s+2)}{(s+1)(s+3)(s+4)}$$

$$F(s) = \frac{A_1}{s+1} + \frac{A_2}{s+3} + \frac{A_3}{s+4}$$

$$A_1 = \lim_{s \rightarrow -1} (s+1) F(s) = \frac{6(s+2)}{(s+3)(s+4)} \Big|_{-1} = \frac{6}{6} = 1$$

$$A_2 = \lim_{s \rightarrow -3} (s+3) F(s) = \frac{6(s+2)}{(s+1)(s+4)} \Big|_{-3} = \frac{-6}{-2} = 3$$

$$A_3 = \lim_{s \rightarrow -4} (s+4) F(s) = \frac{6(s+2)}{(s+1)(s+3)} \Big|_{-4} = \frac{-12}{3} = -4$$

$$F(s) = \frac{1}{s+1} + \frac{3}{s+3} + \frac{-4}{s+4}$$

$$f(t) = (e^{-t} + 3e^{-3t} - 4e^{-4t})u(t) \quad \text{Ans}$$

$$5) F(s) = \frac{10s^2 + 4}{s(s+1)(s+2)^2}$$

$$F(s) = \frac{A_1}{s} + \frac{A_2}{s+1} + \frac{R_1}{(s+2)^2} + \frac{R_2}{s+2}$$

$$A_1 = \frac{10s^2 + 4}{(s+1)(s+2)^2} \Big|_0 = \frac{4}{4} = 1$$

$$A_2 = \frac{10s^2 + 4}{s(s+2)^2} \Big|_{-1} = \frac{14}{-1} = -14$$

$$R_1 = \frac{10s^2 + 4}{s(s+1)} \Big|_{-2} = \frac{44}{2} = 22$$

$$R_2 = \lim_{s \rightarrow -2} \frac{d}{ds} \left(\frac{10s^2 + 4}{s(s+1)} \right) = \lim_{s \rightarrow -2} \frac{s(s+1)20s - (10s^2 + 4) \frac{d}{ds} s(s+1)}{s^2(s+1)^2}$$

$$= \lim_{s \rightarrow -2} \frac{20s^2(s+1) - (10s^2 + 4)(s+s+1)}{s^2(s+1)^2}$$

$$= \frac{-80 - 44(-3)}{4} = \frac{-80 + 132}{4} = \frac{52}{4}$$

= 13

$$F(s) = \frac{1}{s} + \frac{-14}{s+1} + \frac{22}{(s+2)^2} + \frac{13}{s+2}$$

$$f(t) = (1 - 14e^{-t} + 22te^{-2t} + 13e^{-2t})u(t)$$

$$6) F(s) = \frac{10}{(s+3)(s^2+8s+25)}$$

$$s^2+8s+25$$

$$p = \frac{-8 \pm \sqrt{64-100}}{2}$$

$$= \frac{-8 \pm 6i}{2}$$

$$= -4 \pm 3i$$

$$p = -4+3i \quad p^* = -4-3i =$$

$$\frac{10}{(s+3)(s^2+8s+25)} = \frac{A}{s+3} + \frac{B}{s-p} + \frac{B^*}{s-p^*}$$

$$A = \lim_{s \rightarrow -3} \frac{10}{s^2+8s+25} = \frac{10}{9-24+25} = 1$$

$$B = \lim_{s \rightarrow p} \frac{10}{(s+3)(s-p^*)}$$

$$= \frac{10}{(-4+3i+3)(-4+3i+4+3i)}$$

$$= \frac{10s}{(-1+3i)(6i)} = \frac{5}{-3i-9}$$

$$= \frac{5}{-3i-9} \times \frac{-9+3i}{-9+3i} = \frac{15(3+i)}{81+9}$$

$$B = \frac{-3+i}{6} \Rightarrow B' = \frac{-3-i}{6}$$

$$\frac{B}{s-p} + \frac{B^*}{s-p^*} = \frac{-\frac{1}{2} + \frac{i}{6}}{s+4-3i} + \frac{-\frac{1}{2} - \frac{i}{6}}{s+4+3i}$$

$$= \frac{2(-\frac{1}{2})(s+4) + 2(\frac{i}{6})(3i)}{(s+4)^2 + 9}$$

$$= \frac{-s-4-1}{(s+4)^2 + 9}$$

$$= \frac{-s-5}{(s+4)^2 + 9}$$

$$F(s) = \frac{1}{s+3} + \frac{-s-5}{(s+4)^2 + 9}$$

$$= e^{-3t} - \frac{s+4}{(s+4)^2 + 9} - \frac{3}{(s+4)^2 + 9} \times \frac{1}{3}$$

$$= \left(e^{-3t} - e^{-4t} \cos(3t) - \frac{1}{3} e^{-4t} \sin(3t) \right) u(t)$$

$$\frac{s^2+4}{s^2+9} = F(s)$$

$$s^2+9 \overline{) \begin{array}{r} s^2+4 \\ -s^2-9 \\ \hline -5 \end{array}}$$

$$F(s) = 1 - \frac{5}{s^2+9}$$

$$s = \pm 3$$

$$\frac{5}{s^2+9} = \frac{A}{s-3i} + \frac{B}{s+3i}$$

$$A = \lim_{s \rightarrow 3i} \frac{5}{s+3i} = \frac{5}{3i+3i}$$

$$= \frac{5}{6i} \times \frac{6i}{6i} = \frac{5 \cdot 30i}{-366}$$

$$A = -\frac{5i}{6}$$

$$B = \lim_{s \rightarrow -3i} \frac{5}{s-3i} = \frac{5}{-3i-3i}$$

$$= -\frac{5}{6i} \times \frac{6i}{6i} = \frac{5i}{6}$$

$$F(s) = 1 - \left(\frac{-\frac{5i}{6}}{s-3i} + \frac{\frac{5i}{6}}{s+3i} \right)$$

$$= 1 + \frac{5i}{6} \left(\frac{1}{s-3i} \right) - \frac{5i}{6} \frac{1}{s+3i}$$

$$= 1 + \frac{5i}{6} \left[\frac{8+3i - s+3i}{s^2+9} \right]$$

$$= 1 + \frac{5i}{6} \left(\frac{6i}{s^2+9} \right)$$

F(s)

$$F(s) = 1 - \frac{5}{s^2+9}$$

$$= 1 - \frac{5}{3} \frac{3}{s^2+9}$$

$$f(t) = \delta(t) - \frac{5}{3} \sin(3t) u(t)$$

$$b) F(s) = \frac{5e^{-bs}}{s^2+s+1}$$

$$s^2+s+1=0$$

$$\frac{-1 \pm \sqrt{1-4}}{2} = \frac{-1 \pm 3i}{2}$$

$$-P = -\frac{1}{2} + \frac{3}{2}i, \quad -P^* = -\frac{1}{2} - \frac{3}{2}i$$

$$\frac{5e^{-bs}}{s^2+s+1} = \frac{A}{s-P} + \frac{A^*}{s-P^*}$$

$$A = \lim_{s \rightarrow P} \frac{5e^{-bs}}{s + \frac{1}{2} + \frac{3}{2}i}$$

$$P = -\frac{1}{2} + \frac{3}{2}i$$

$$P^* = -\frac{1}{2} - \frac{3}{2}i$$

$$= \frac{5e^{-bs}}{3i} \times \frac{3i}{3i} = \frac{5e^{-bs}}{-93} = -\frac{5}{3} e^{-bs} i$$

$$A^* = \lim_{s \rightarrow P^*} \frac{5e^{-bs}}{s + \frac{1}{2} - \frac{3}{2}i}$$

(s+6)

G(s)

$$A^* = \frac{5e^{3(1+3i)}}{-\frac{1}{2} - \frac{3}{2}i + \frac{1}{2} - \frac{3}{2}i}$$

$$= \frac{5e^{3(1+3i)}}{-3i} \times \frac{3i}{3i}$$

$$= \frac{5}{3} e^{3(1+3i)} i$$

$$F(s) = \frac{-\frac{5}{3} e^{3(1-3i)} i}{s + \frac{1}{2} - \frac{3}{2}i} + \frac{\frac{5}{3} e^{3(1+3i)} i}{s + \frac{1}{2} + \frac{3}{2}i}$$

$$= \frac{5}{3} i \left[\frac{-e^{3(1-3i)}}{(s + \frac{1}{2})^2 + \frac{9}{4}} \right]$$

$$G(s) = \frac{+\frac{5}{3}i}{s + \frac{1}{2} - \frac{3}{2}i} + \frac{\frac{5}{3}i}{s + \frac{1}{2} + \frac{3}{2}i}$$

$$= \frac{2(-\frac{5}{3}i)(\frac{3}{2}i)}{(s + \frac{1}{2})^2 + \frac{9}{4}}$$

$$= \frac{5}{(s + \frac{1}{2})^2 + \frac{9}{4}}$$

$$= \frac{5}{\frac{3}{2}} \cdot \frac{\frac{3}{2}}{(s + \frac{1}{2})^2 + \frac{9}{4}}$$

$$= e^{\frac{1}{2}t} \frac{10}{3} \sin\left(\frac{3}{2}t\right) u(t)$$

$$e^{-bs} G(s) = \frac{10}{3} e^{-\frac{1}{2}(t-b)} \sin\left(\frac{3}{2}(t-b)\right) u(t-b)$$

$$9) F(s) = \sum_{n=0}^{\infty} e^{-nTs}$$

$$G(s) = \sum_{n=0}^{\infty} 1$$

$$g(t) = \delta(t)$$

$$e^{-nTs} G(s) = \delta(t - nT) \quad \Delta$$

$$10) F(s) = \sum_{n=0}^{\infty} [e^{-2nTs} - e^{-(2n+1)Ts}]$$

$$F(s) = (-1)^n \sum_{n=0}^{\infty} e^{-nTs}$$

$$f(t) = \sum_{n=0}^{\infty} (-1)^n \delta(t - nT) \quad \Delta$$

$$\sum_{n=0}^{\infty} (-n)$$

$$\sum_{n=0}^{\infty} (2n - (2n+1))$$

$$\sum_{n=0}^{\infty} (-1)$$

$$\sum_{n=0}^{\infty} (2^{2n} - 2^{2n+1})$$

$$= \frac{5}{(s + \frac{1}{2})^2 + \frac{3}{4}} \cdot e^{-bs}$$

$$= \frac{2}{\sqrt{3}} \frac{5 \cdot \frac{\sqrt{3}}{2}}{(s + \frac{1}{2})^2 + (\frac{\sqrt{3}}{2})^2} e^{-bs}$$

$$= \frac{10}{\sqrt{3}} \sin\left(\frac{\sqrt{3}}{2}(t-b)\right) e^{-\frac{1}{2}(t-b)} u(t-b)$$

$$F(s) = \frac{1}{(s+1)(s+3)^3}$$

$$\Rightarrow f(t) = ?$$

$$F(s) = \frac{1}{(s+1)(s+3)^3}$$

$$F(s) = \frac{A}{s+1} + \frac{B_1}{s+3} + \frac{B_2}{(s+3)^2} + \frac{B_3}{(s+3)^3}$$

$$A = \lim_{s \rightarrow -1} \frac{1}{(s+3)^3} = \frac{1}{8}$$

$$B_3 = \lim_{s \rightarrow -3} \frac{1}{s+1} = -\frac{1}{2}$$

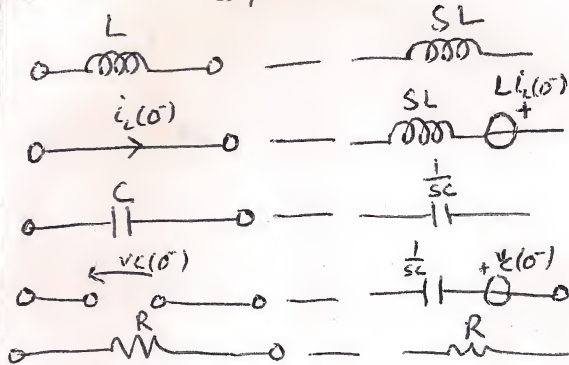
$$B_2 = \lim_{s \rightarrow -3} \frac{d}{ds} \left(\frac{1}{s+1} \right) = -$$

$$B_1 = \lim_{s \rightarrow -3} \frac{1}{2!} \frac{d^2}{ds^2} \left(\frac{1}{s+1} \right)$$

$$(s+3)^3 F(s) = G(s) = G(-3) + G'(-3)(s+3) + \frac{G''(-3)}{2!} (s+3)^2$$

BY PROFESSOR

SYMBOLIC CKT:



gf $v_C(0^-)$, \forall constant values $\rightarrow \frac{v_C(0^-)}{s}$
 $3V \rightarrow \frac{3}{s}$

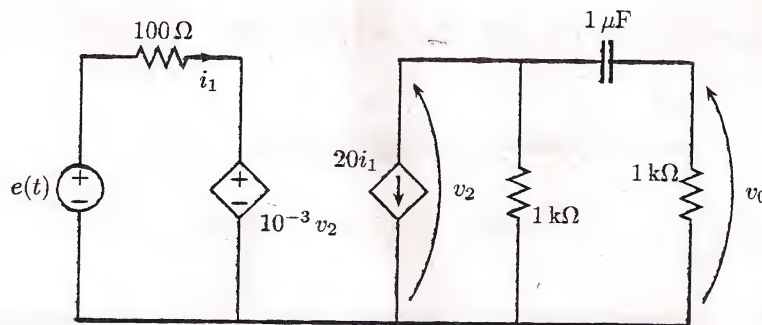
gf $i_L(0^-)$, \forall constant values $\rightarrow \frac{i_L(0^-)}{s}$
 $4A \rightarrow \frac{4}{s}$

Chapter 11

Laplace transform II: application to circuits

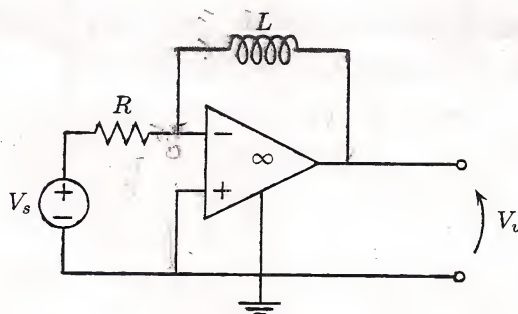
Exercise 11.1

Calculate $H(s) = \frac{V_0(s)}{E(s)}$ and $h(t)$.



Exercise 11.2

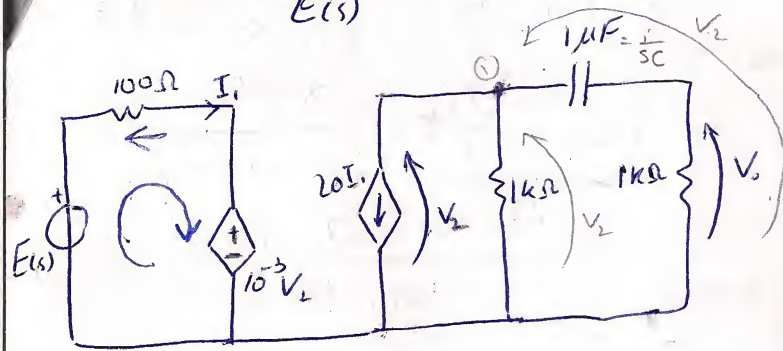
Evaluate $V_u(V_s)$, then repeat replacing L with C .



LAPLACE TRANSFORM II

Application to circuits

1. $H(s) = \frac{V_o(s)}{E(s)}$, $h(t) = ?$



KCL @ ①

$$20I_1 + \frac{V_L}{10^3} + \frac{V_L}{10^3 + \frac{10^6}{s}} = 0$$

$$I_1 = \frac{E(s) - 10^{-3}V_L}{100} \text{ put in above}$$

$$20 \left(\frac{E(s) - 10^{-3}V_L}{100} \right) + \frac{V_L}{10^3} + \frac{V_L}{10^3 + \frac{10^6}{s}} = 0$$

$$\frac{E(s)}{5} - \frac{10^{-3}}{5}V_L + \frac{V_L}{10^3} + \frac{V_L}{10^3 + \frac{10^6}{s}} = 0$$

$$V_L \left(\frac{1}{10^3} + \frac{1}{10^3 + \frac{10^6}{s}} - \frac{10^{-3}}{5} \right) = -\frac{E(s)}{5}$$

$$V_L \left(\frac{1}{10^3} + \frac{s}{5s10^3 + 10^6} - \frac{10^{-3}}{5} \right) = -\frac{E(s)}{5}$$

$$V_L \left(\frac{5(s10^3 + 10^6) + 5s10^3 - 10(s10^3 + 10^6)}{5 \cdot 10^3 (s10^3 + 10^6)} \right) = -\frac{E(s)}{5}$$

$$V_L \left(\frac{4(s10^3 + 10^6) + 5s10^3}{10^3 (s10^3 + 10^6)} \right) = -E(s)$$

$$V_L = \frac{-E(s) 10^3 (s10^3 + 10^6)}{4(s10^3 + 10^6) + 5s10^3}$$

$$V_o = \frac{10^3}{10^3 + \frac{10^6}{s}} V_L$$

$$V_o = \frac{s10^3}{s10^3 + 10^6} \left(\frac{-E(s) 10^3 (s10^3 + 10^6)}{4(s10^3 + 10^6) + 5s10^3} \right)$$

$$V_o = \frac{-E(s) s10^6}{4(s10^3 + 10^6) + 5s10^3}$$

$$H(s) = \frac{-s10^6}{4(s10^3 + 10^6) + 5s10^3}$$

$$= \frac{-s10^6}{4s10^3 + 410^6 + 5s10^6}$$

$$= \frac{-s10^6}{9s10^3 + 410^6} = \frac{-s10^3}{9s + 4 \cdot 10^3}$$

$$= \frac{-s10^3}{9(s + \frac{4 \cdot 10^3}{9})}$$

$$H(s) = -\frac{10^3}{9} \left(\frac{s}{s + \frac{4}{9} \times 10^3} \right)$$

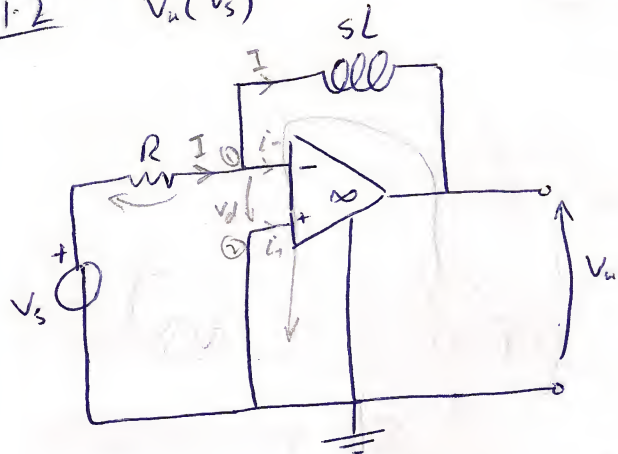
$$s + \frac{4}{9} \times 10^3 \sqrt{\frac{s}{s + \frac{4}{9} \times 10^3}} = \frac{1}{-\frac{4}{9} \times 10^3}$$

$$= -\frac{10^3}{9} \left(1 + \frac{-\frac{4}{9} \times 10^3}{s + \frac{4}{9} \times 10^3} \right)$$

$$H(s) = -\frac{10^3}{9} + \frac{\frac{4}{81} 10^6}{s + \frac{4}{9} \times 10^3}$$

$$h(t) = -\frac{10^3}{9} \delta(t) + \frac{4 \times 10^6}{81} e^{-\frac{4}{9} \times 10^3 t} u(t)$$

11.2 $V_u(V_s)$



$$I = \frac{V_s}{R}$$

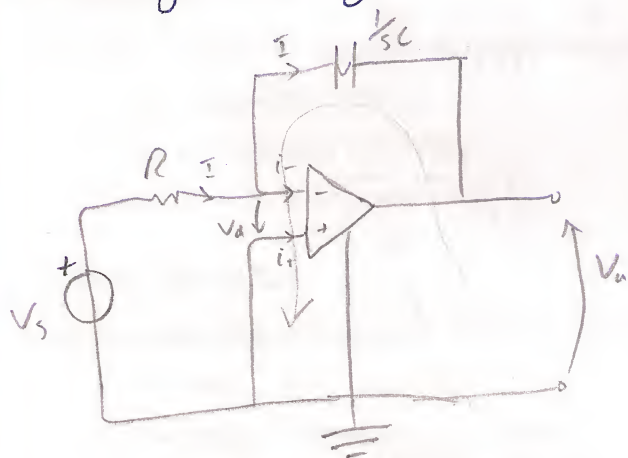
$$V_s = I R$$

$$V_u + I S L + V_d = 0$$

$$V_u = -I S L$$

$$V_u = -\frac{S L}{R} V_s \quad \text{Ans}$$

Replacing L by C



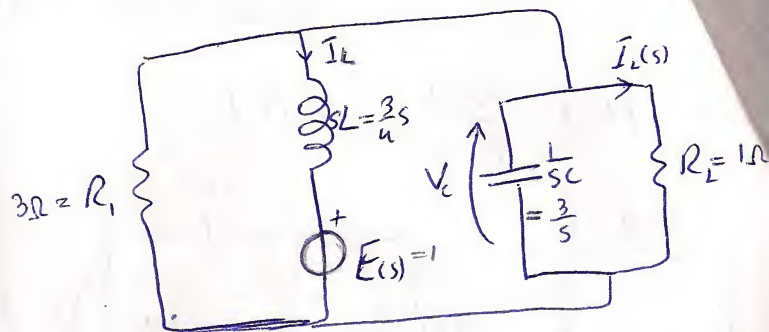
$$I = \frac{V_s}{R}$$

$$\text{KVL} \quad V_u + \frac{I}{S C} + V_d = 0$$

$$V_u = -\frac{I}{S C}$$

$$V_u = -\frac{1}{S C R} V_s \quad \text{Ans}$$

11.3 impulse response of i_L



$$V_c = \frac{\frac{E(s)}{\frac{3s}{4}}}{\frac{1}{3} + \frac{4}{3s} + \frac{s}{3} + 1} = \frac{\frac{E(s) 4}{3s}}{\frac{s+4+s^2+3s}{3s}}$$

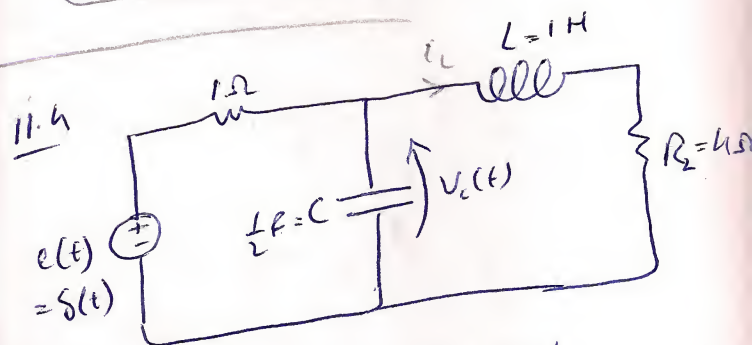
$$V_c = \frac{4 E(s)}{s^2 + 4s + 4}$$

$$I_L(s) = \frac{4 E(s)}{(s+2)^2} \quad \checkmark$$

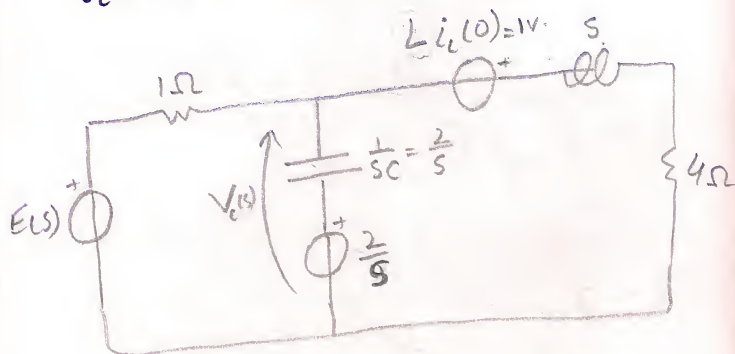
$$H(s) = \frac{4}{(s+2)^2}$$

$$h(t) = 4t e^{-2t} u(t)$$

$$H(s) = \frac{I_2(s)}{E(s)}; E(s) = 1$$



$$i_L(0) = 1 \text{ A}, \quad V_c(0) = 2 \text{ V}$$



$$\begin{aligned}
 I(s) &= \frac{\frac{2/s}{2/s} + \frac{E(s)}{1} - \frac{1}{s+4}}{1 + \frac{s}{2} + \frac{1}{s+4}} \\
 &= \frac{1 + 1 - \frac{1}{s+4}}{\frac{2(s+4) + s(s+4) + 2}{2(s+4)}} \\
 &= \frac{\frac{s+4 + s+4 - 1}{s+4}}{\frac{2s+8 + s^2+4s+2}{2(s+4)}} = \frac{2(2s+7)}{s^2+6s+10}
 \end{aligned}$$

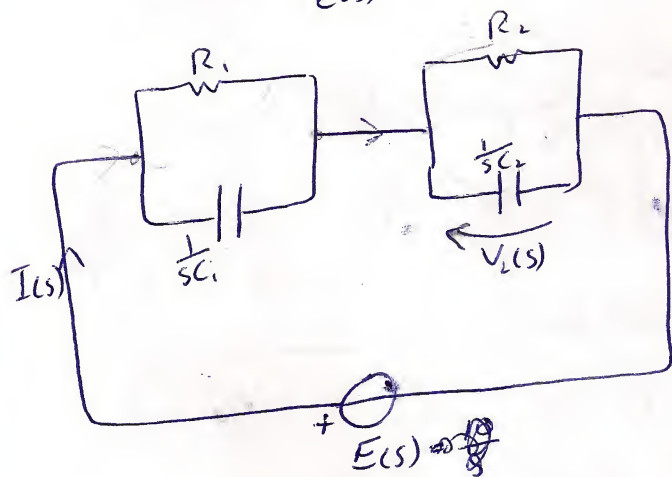
$$V_c(s) = \frac{4s+14}{s^2+2(3)s+(3)^2+1} = \frac{4s+12+2}{(s+3)^2+(1)^2}$$

$$V_c(s) = \frac{4(s+3)}{(s+3)^2+(1)^2} + \frac{2}{(s+3)^2+(1)^2}$$

$$V_c(t) = [4e^{-3t} \cos(t) + 2e^{-3t} \sin(t)] u(t)$$

11.5 $H(s) = \frac{I(s)}{E(s)}$

$$H_2(s) = \frac{V_L(s)}{E(s)}$$



$$\frac{R_1 \frac{1}{sC_1}}{R_1 + \frac{1}{sC_1}} = \frac{\frac{R_1/sC_1}{sR_1C_1+1}}{sC_1} = \frac{R_1}{1+sR_1C_1}$$

$$\frac{R_2 \frac{1}{sC_2}}{R_2 + \frac{1}{sC_2}} = \frac{\frac{R_2/sC_2}{sR_2C_2+1}}{sC_2} = \frac{R_2}{1+sR_2C_2}$$

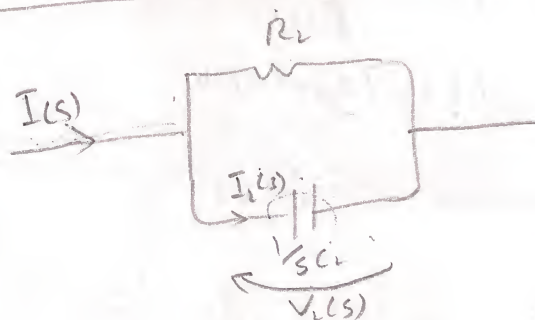
$$\begin{aligned}
 Z_{eq} &= \frac{R_1}{1+sR_1C_1} + \frac{R_2}{1+sR_2C_2} \\
 &= \frac{R_1(1+sR_2C_2) + R_2(1+sR_1C_1)}{(1+sR_1C_1)(1+sR_2C_2)}
 \end{aligned}$$

$$I(s) = \frac{E(s)}{Z_{eq}}$$

$$I(s) = \frac{E(s)(1+sR_1C_1)(1+sR_2C_2)}{R_1 + sR_1R_2C_2 + R_2 + sR_1R_2C_1}$$

$$H(s) = \frac{I(s)}{E(s)}$$

$$H(s) = \frac{(1+sR_1C_1)(1+sR_2C_2)}{R_1 + R_2 + sR_1R_2(C_1 + C_2)}$$



$$I_2(s) = \frac{R_2}{R_2 + \frac{1}{sC_2}} I(s)$$

$$= \frac{sR_2C_2}{1 + sC_2R_2} \left(\frac{E(s)(1+sR_1C_1)(1+sR_2C_2)}{R_1 + R_2 + sR_1R_2(C_1 + C_2)} \right)$$

$$I_2(s) = \frac{E(s) sR_2C_2 (1+sR_1C_1)}{R_1 + R_2 + sR_1R_2(C_1 + C_2)}$$

$$V_L(s) = \frac{E(s) sR_2R_2 (1+sR_1C_1)}{R_1 + R_2 + sR_1R_2(C_1 + C_2)} \propto \frac{1}{sC_2}$$

$$V_L(s) = \frac{E(s)(1+sR_1C_1)R_2}{R_1 + R_2 + sR_1R_2(C_1 + C_2)}$$

$$H_2(s) = \frac{(1+sR_1C_1)R_2}{R_1 + R_2 + sR_1R_2(C_1 + C_2)}$$

For relationship

$$(1 + sC_1 R_1) R_L = 0$$

$$1 + sC_1 R_1 = 0$$

$$s = -\frac{1}{C_1 R_1} \checkmark$$

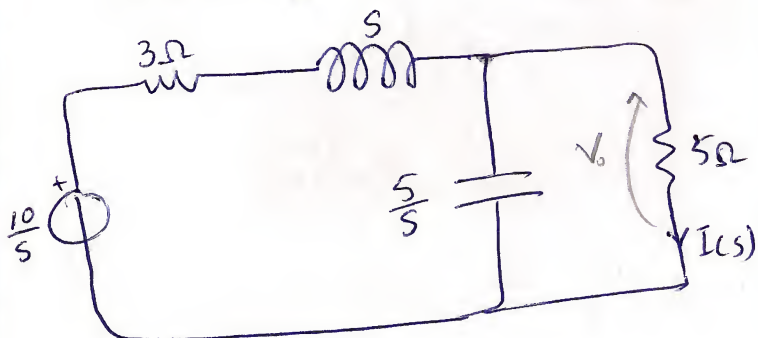
$$1 + s \frac{R_1 R_2}{R_1 + R_2} (C_1 + C_2) = 0$$

$$+ \frac{R_1 R_2}{C_1 R_1 (R_1 + R_2)} (C_1 + C_2) = 0$$

$$\boxed{\frac{R_2}{R_1 + R_2} = \frac{C_1}{C_1 + C_2}} \quad \underline{\underline{A}}$$

11.6 $H(s) = \frac{I(s)}{E(s)}$, $i(t)$

$$e(t) = 10\delta(t) \text{ V} \quad E(s) = \frac{10}{s}$$



$$V_o(s) = \frac{10/s}{3+s}$$

$$= \frac{10/s}{\frac{1}{s+3} + \frac{s}{5} + \frac{1}{5}}$$

$$= \frac{10 \cdot 5}{s(5 + s(3+s) + s+3)}$$

$$= \frac{50}{s(s^2 + 4s + 8)}$$

$$I(s) = \frac{V_o}{5} = \frac{10}{s(s^2 + 4s + 8)}$$

$$H(s) = \frac{\cancel{10/s}}{\cancel{s^2 + 4s + 8}}$$

$$H(s) = \frac{1}{s^2 + 2(s)2 + (2)^2 + 4}$$

$$\boxed{H(s) = \frac{1}{(s+2)^2 + 4}}$$

or

$$\boxed{H(s) = \frac{1}{s^2 + 4s + 8}}$$

$$I(s) = \frac{10}{s(s^2 + 4s + 8)}$$

$$\frac{10}{s(s^2 + 4s + 8)} = \frac{A}{s} + \frac{Bs + C}{s^2 + 4s + 8}$$

$$10 = A(s^2 + 4s + 8) + (Bs + C)s$$

$$s=0$$

$$10 = 8A \Rightarrow$$

$$\boxed{A = \frac{5}{4}}$$

$$s=1$$

$$10 = \frac{5}{4}(13) + (B+C)$$

$$B+C = 10 - \frac{65}{4}$$

$$B+C = -\frac{25}{4} \quad \text{--- (1)}$$

$$s=-1 \quad 10 = \frac{5}{4}(-1-4+8) + B - C$$

$$10 = \frac{25}{4} + B - C$$

$$B - C = \frac{15}{4} \quad \text{--- (2)}$$

$$\textcircled{1} + \textcircled{2}$$

$$2B = -\frac{10}{4}$$

$$\boxed{B = -\frac{5}{4}}$$

$$C = -\frac{25}{4} + \frac{5}{4}$$

$$\boxed{C = -5}$$

$$s) = \frac{5/4}{s} + \frac{-\frac{5}{4}s - 5}{(s+2)^2 + 4}$$

$$= \frac{5}{4} \frac{1}{s} + \frac{-\frac{5s-20}{4}}{(s+2)^2 + 4}$$

$$= \frac{5}{4} \frac{1}{s} - \frac{5}{4} \left(\frac{s+4}{(s+2)^2 + 4} \right)$$

$$= \frac{5}{4} \frac{1}{s} - \frac{5}{4} \left(\frac{(s+2)+2}{(s+2)^2 + 4} \right)$$

$$= \frac{5}{4} \frac{1}{s} - \frac{5}{4} \left[\frac{s+2}{(s+2)^2 + 4} + \frac{2}{(s+2)^2 + 4} \right]$$

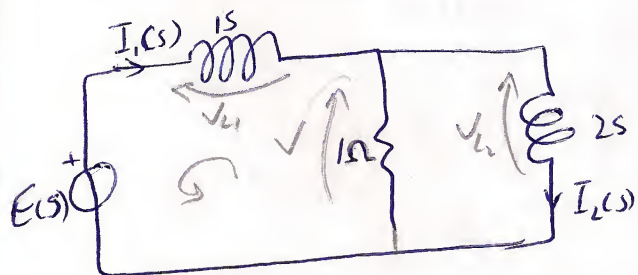
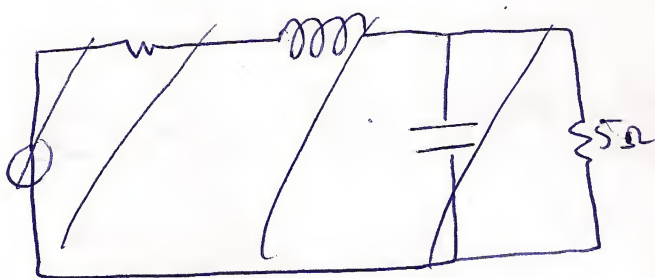
$$i(t) = \frac{5}{4} \left[u(t) - (e^{-2t} \cos(2t) + e^{-2t} \sin(2t)) u(t) \right]$$

$$i(t) = \frac{5}{4} \left[1 - e^{-2t} (\cos(2t) + \sin(2t)) \right] u(t)$$

11.7 $H_1(s) = \frac{I_1(s)}{E(s)}$, $H_2(s) = \frac{I_2(s)}{E(s)}$

$$e(t) = 0V$$

$$i_1(0) = 2A, \quad i_2(0) = 1A$$



$$V = \frac{E(s)/s}{\frac{1}{s} + 1 + \frac{1}{2s}}$$

$$V = \frac{E(s)/s}{\frac{2+2s+1}{2s}} = \frac{2E(s)}{2s+3}$$

$$V + V_{L1} - E(s) = 0$$

$$V_{L1} = E(s) - \frac{2E(s)}{2s+3}$$

$$V_{L1} = \frac{E(s)(2s+3) - 2E(s)}{2s+3}$$

$$V_{L1} = \frac{E(s)(2s+3-2)}{2s+3}$$

$$I_1(s) = \frac{E(s)(2s+1)}{s(2s+3)} \quad ?$$

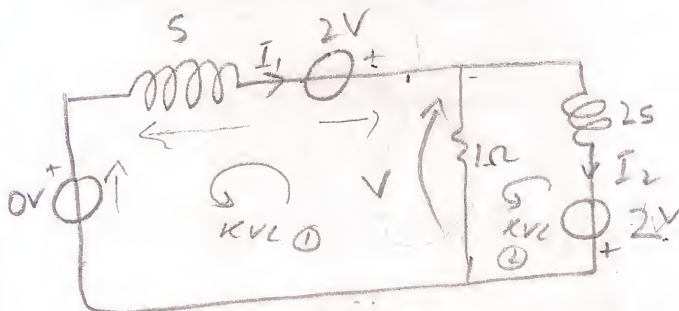
$$H_1(s) = \frac{2(s+1/2)}{2s(s+3/2)}$$

$$H_1(s) = \frac{s+1/2}{s(s+3/2)}$$

$$V_{L2} = V$$

$$I_2(s) = \frac{2E(s)}{(2s+3)} \cdot \frac{1}{2s}$$

$$H_2(s) = \frac{1}{2s(s+3/2)}$$



$$V = \frac{2/s - 1/s}{\frac{1}{s} + 1 + \frac{1}{2s}} = \frac{2-1}{\frac{2+2s+1}{2s}}$$

$$V = \frac{2}{2s+3} V$$

KVL ①

$$I_1 s - 2 + V = 0$$

$$I_1(s) s - 2 + \frac{2}{2s+3} = 0$$

$$I_1(s) = \frac{4s+6-2}{2s+3}$$

$$I_1(s) = \frac{4(s+1)}{2s(s+\frac{3}{2})}$$

$$I_1(s) = \frac{2(s+1)}{s(s+\frac{3}{2})}$$

$$\frac{2(s+1)}{s(s+\frac{3}{2})} = \frac{A}{s} + \frac{B}{s+\frac{3}{2}}$$

$$2(s+1) = A(s+\frac{3}{2}) + B(s)$$

$$\frac{s=0}{2} = \frac{3}{2} A \Rightarrow \boxed{A = \frac{4}{3}}$$

$$s = -\frac{3}{2}$$

$$2(-\frac{3}{2}+1) = B(-\frac{3}{2})$$

$$-1 = B(-\frac{3}{2}) \Rightarrow \boxed{B = \frac{2}{3}}$$

$$I_1(s) = \frac{4/3}{s} + \frac{2/3}{s+\frac{3}{2}}$$

$$i_1(t) = \left(\frac{4}{3} + \frac{2}{3} e^{-\frac{3}{2}t} \right) u(t) \text{ A}$$

KVL ②

$$I_2(s) - 2 - V = 0$$

$$I_2(s) = 2 + \frac{2}{2s+3}$$

$$I_2(s) = \frac{4s+6+2}{2s(2s+3)}$$

$$I_2(s) = \frac{s+2}{s(s+\frac{3}{2})}$$

$$\frac{s+2}{s(s+\frac{3}{2})} = \frac{A}{s} + \frac{B}{s+\frac{3}{2}}$$

$$(s+2) = A(s+\frac{3}{2}) + B(s)$$

$$s=0$$

$$2 = \frac{3}{2} A \Rightarrow \boxed{A = \frac{4}{3}}$$

$$s = -\frac{3}{2}$$

$$-\frac{3}{2} + 2 = -B \frac{3}{2}$$

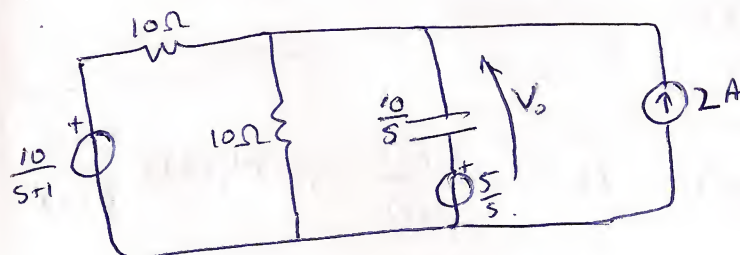
$$\frac{1}{2} = -B \frac{3}{2} \Rightarrow \boxed{B = -\frac{1}{3}}$$

$$I_2(s) = \frac{4/3}{s} + \frac{-1/3}{s+\frac{3}{2}}$$

$$i_2(t) = \left(\frac{4}{3} - \frac{1}{3} e^{-\frac{3}{2}t} \right) u(t) \text{ A}$$

11.8 $V_o(0) = 5 \text{ V}$

$$V_o(t) = ?$$



$$V_o(s) = \frac{10/s+1}{s+0} + \frac{5/s}{10/s} + 2$$

$$\frac{1}{10} + \frac{1}{10} + \frac{5}{10}$$

$$= \frac{\frac{1}{s+1} + \frac{1}{2} + 2}{\frac{2+s}{10}} = \frac{2+s+1+4s+4}{2(s+1)} = \frac{2+s}{10}$$

$$= \frac{5s+7}{2(s+1)} \times \frac{s}{2+s}$$

$$V_o(s) = \frac{5(5s+7)}{(s+1)(s+2)}$$

$$\frac{5(5s+7)}{(s+1)(s+2)} = \frac{A}{s+1} + \frac{B}{s+2}$$

$$5(5s+7) = A(s+2) + B(s+1)$$

$$(2) = A \Rightarrow A = 10$$

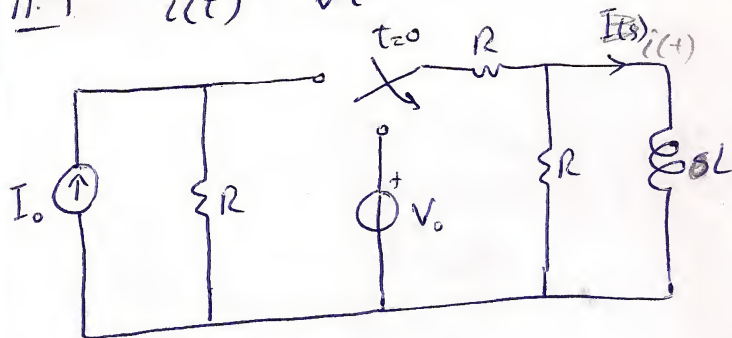
$$s = -2$$

$$5(-3) = B(-1) \Rightarrow B = 15$$

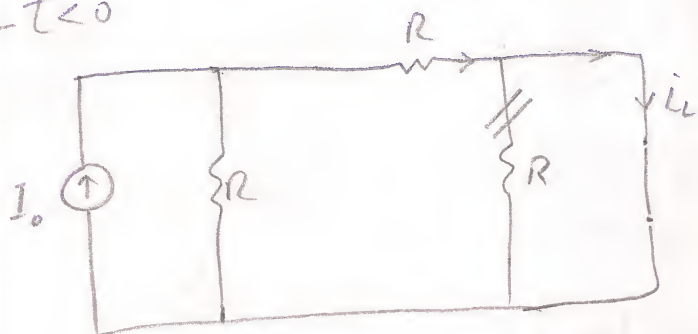
$$V_o(s) = \frac{10}{s+1} + \frac{15}{s+2}$$

$$V_o(t) = (10e^{-t} + 15e^{-2t})u(t) \text{ V}$$

11.9 $i(t)$ $\forall t$

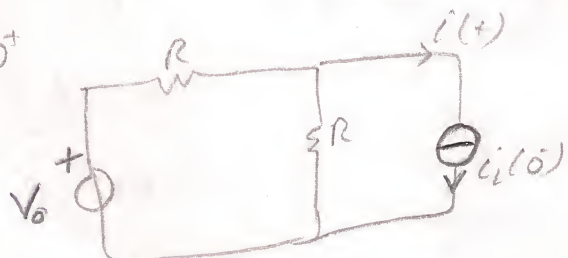


$-t < 0$



$$i_L(0^-) = \frac{R}{2R} I_0 \Rightarrow i_L(0^-) = \frac{I_0}{2} \text{ A}$$

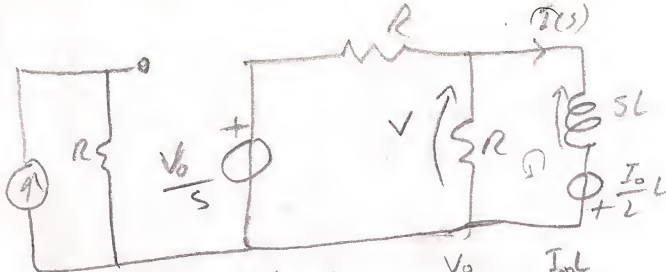
$-t \rightarrow 0^+$



$$i_L(0^+) = i_L(0^-) = \frac{I_0}{2}$$

(Steady-state)

$t > 0$



$$V = \frac{\frac{V_0/s}{R} - \frac{I_0 L}{2/sL}}{\frac{1}{R} + \frac{1}{R} + \frac{1}{sL}} = \frac{\frac{V_0}{sR} - \frac{I_0 L}{2sL}}{\frac{2sL + R}{RsL}}$$

$$V = \frac{\frac{2V_0 L - I_0 L R}{2sR L}}{\frac{2sL + R}{RsL}} = \frac{L(2V_0 - I_0 R)}{2(2sL + R)}$$

$$V = \frac{L(2V_0 - I_0 R)}{2(R + 2sL)}$$

KVL

$$I(s) sL = \frac{I_0 L}{2} + \frac{L}{2} \left(\frac{2V_0 - I_0 R}{R + 2sL} \right)$$

$$I(s) = \frac{K}{2sL} \left(I_0 + \frac{2V_0 - I_0 R}{R + 2sL} \right)$$

$$I(s) = \frac{I_0 R + I_0 2sL + 2V_0 - I_0 R}{2s(R + 2sL)}$$

$$I(s) = \frac{I_0 2sL + 2V_0}{2s(R + 2sL)}$$

$$I(s) = \frac{2(V_0 + I_0 sL)}{2Ls(s + \frac{R}{2L})}$$

$$V_0 + I_0 sL = A(s + \frac{R}{2L}) + B(s)$$

$$-s = 0$$

$$V_0 = A \frac{R}{2L} \Rightarrow A = \frac{2V_0 L}{R} \cdot \frac{1}{2L}$$

$$A = \frac{V_0}{R}$$

$$-s = -\frac{R}{2L}$$

$$-\frac{R}{2L} B = V_0 - \frac{I_0 L \cdot R}{2L}$$

$$B = -\frac{2V_0 - I_0 R}{R} \cdot \frac{2L}{R}$$

$$B = -\frac{L}{R} \left(\frac{2V_0 - I_0 R}{2L} \right)$$

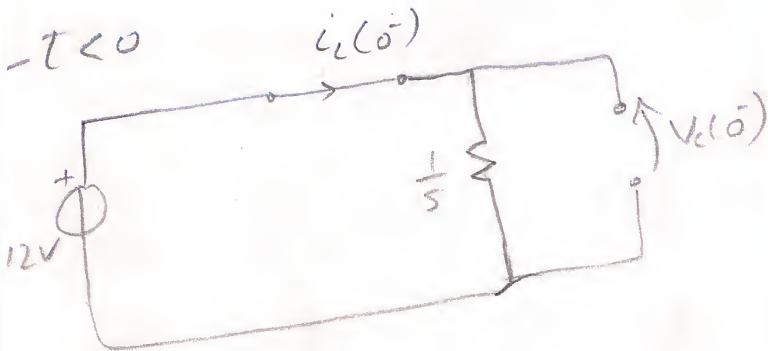
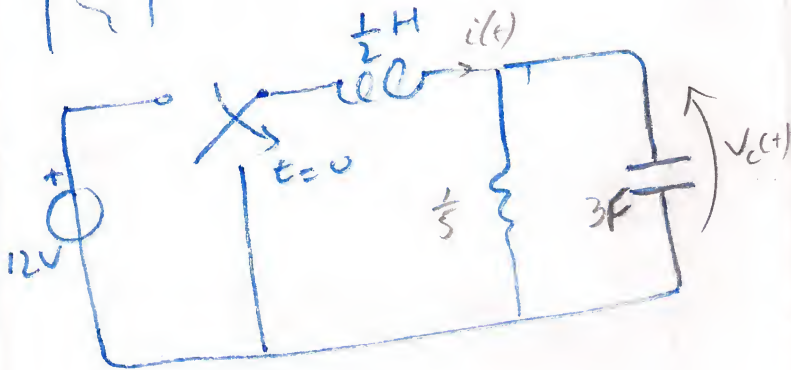
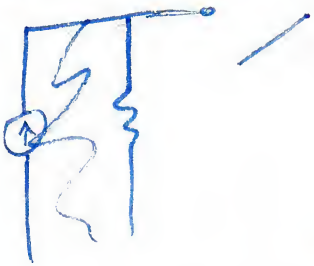
$$B = \frac{I_0 R - 2V_0}{2R}$$

$$I(s) = \frac{2V_0/L/R}{s} + \frac{-\frac{L}{R}(2V_0 - I_0 R)}{s + \frac{R}{2L}}$$

$$I(s) = \frac{V_0/R}{s} + \frac{(I_0 R - 2V_0)/2R}{s + \frac{R}{2L}}$$

$$i(t) = \frac{V_0}{R} u(t) + \left(\frac{I_0}{2} - \frac{V_0}{R} \right) e^{-\frac{R}{2L}t} u(t)$$

11.10



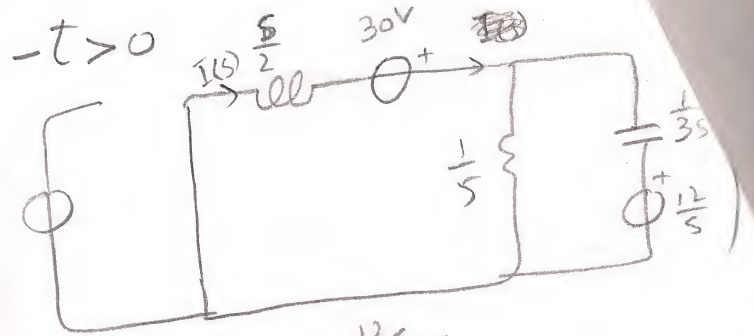
$$V_c(0^-) = 12V$$

$$i_L(0^-) = \frac{12}{1/5} = 60A$$

$-t \rightarrow 0^+$

$$V_c(0^+) = V_c(0^-) = 12V$$

$$i_L(0^+) = i_L(0^-) = 60A$$



$$V_c(s) = \frac{\frac{30}{s/2} + \frac{12/s}{1/3s}}{\frac{1}{s/2} + \frac{1}{1/5} + \frac{1}{1/3s}}$$

$$V_c(s) = \frac{\frac{60}{s} + 36}{\frac{2}{s} + 5 + 3s} = \frac{\frac{60 + 36s}{s}}{\frac{2 + 5s + 3s^2}{s}}$$

$$V_c(s) = \frac{36s + 60}{3s^2 + 5s + 2}$$

$$= \frac{36s + 60}{3s^2 + 3s + 2s + 2} = \frac{36s + 60}{3s(s+1) + 2(s+1)}$$

$$V_c(s) = \frac{36s + 60}{(s+1)(3s+2)}$$

$$\frac{36s + 60}{(s+1)(3s+2)} = \frac{A}{s+1} + \frac{B}{3s+2}$$

$$36s + 60 = A(3s+2) + B(s+1)$$

$$-s = -1 \quad -36 + 60 = -A \Rightarrow A = -24$$

$$-s = -\frac{2}{3} \quad -24 + 60 = B\left(-\frac{2}{3} + 1\right) \Rightarrow \frac{B}{3} = 36 \Rightarrow B = 108$$

$$V_c(s) = \frac{-24}{s+1} + \frac{\frac{108}{3}}{s + \frac{2}{3}}$$

$$V_c(t) = \left(-24e^{-t} + 36e^{-\frac{2}{3}t} \right) u(t) V$$

$$(s) \frac{s}{2} + 30 - \frac{36s + 60}{(s+1)(3s+2)} = 0$$

$$I(s) \frac{s}{2} = 30 - \frac{36s + 60}{(s+1)(3s+2)}$$

$$I(s) = \frac{60}{s} - \frac{72s + 120}{s(s+1)(3s+2)}$$

$$I(s) = \frac{180s^2 + 300s + 120 - 72s - 120}{s(s+1)(3s+2)}$$

$$I(s) = \frac{180s^2 + 228s}{s(s+1)(3s+2)}$$

$$I(s) = \frac{3s(60s + 76)}{3s(s+1)(s+\frac{2}{3})}$$

$$I(s) = \frac{60s + 76}{(s+1)(s+\frac{2}{3})}$$

$$\frac{60s + 76}{(s+1)(s+\frac{2}{3})} = \frac{A}{s+1} + \frac{B}{s+\frac{2}{3}}$$

$$60s + 76 = A(s + \frac{2}{3}) + B(s+1)$$

$$-s = -1$$

$$-60 + 76 = A(-\frac{1}{3}) \Rightarrow -\frac{A}{3} = 16$$

$$A = -48$$

$$-s = -\frac{2}{3}$$

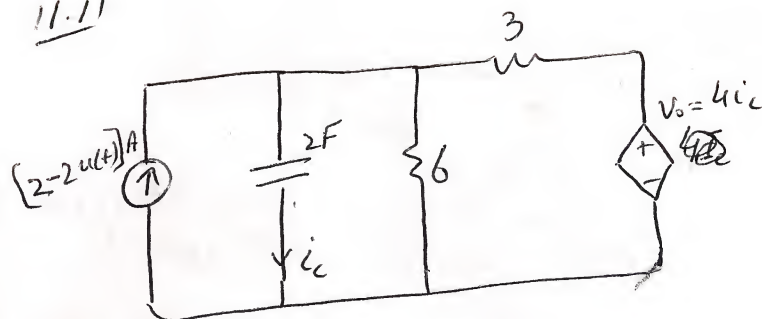
$$-40 + 76 = 0 + B(\frac{1}{3}) \Rightarrow \frac{B}{3} = 36$$

$$B = 108$$

$$I(s) = \frac{-48}{s+1} + \frac{108}{s+\frac{2}{3}}$$

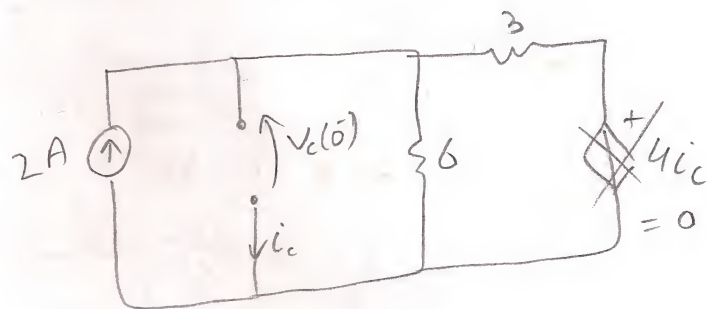
$$i(t) = (-48e^{-t} + 108e^{-\frac{2}{3}t})u(t) \text{ A}$$

11.11

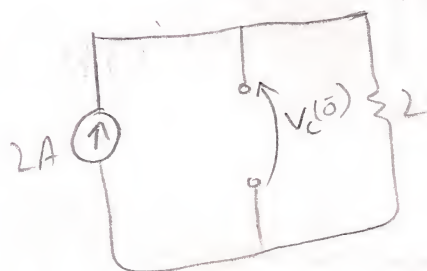


For initial conditions

$$t < 0 \quad u(t) = 0$$

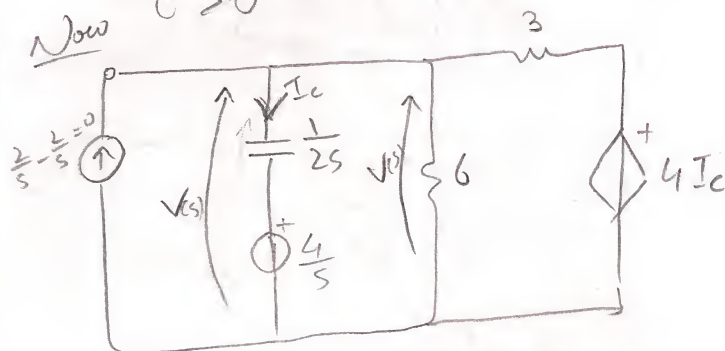


$$i_c = 0$$



$$v_c(0) = 2 \times 2 = 4 \text{ V}$$

Now $t > 0$



$$V = \frac{\frac{4}{s} + \frac{4i_c}{3}}{2s + \frac{1}{6} + \frac{1}{3}} = \frac{3 \times 8 + 4i_c}{12s + 1 + 2}$$

$$V = \frac{8(i_c + 6)}{3(4s + 1)}$$

KVL

$$\frac{I_c}{2s} + \frac{4}{s} + \frac{2(I_c + 6)}{3(4s+1)} = 0$$

$$I_c \left(\frac{1}{2s} - \frac{2}{12s+3} \right) = -\frac{4}{s} + \frac{48}{12s+3}$$

$$I_c \left(\frac{12s+3-16s}{2s(12s+3)} \right) = \frac{-48s-12+48s}{s(12s+3)}$$

$$I_c \left(\frac{-4s+3}{2} \right) = -12$$

$$I_c = \frac{-24}{-4s+3}$$

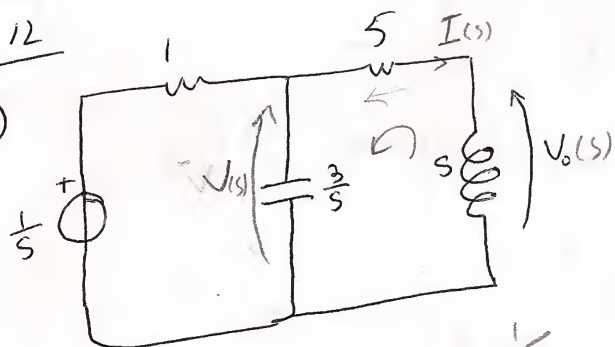
$$I_c = \frac{6}{s - \frac{3}{4}}$$

$$i_c(t) = 6e^{\frac{3}{4}t} u(t) \text{ A}$$

$$v_o(t) = 24e^{\frac{3}{4}t} u(t) \text{ V}$$

11.12

a)



$$V(s) = \frac{\frac{1}{s}}{1 + \frac{s}{3} + \frac{1}{s+5}} = \frac{\frac{1}{s}}{\frac{3s+15+s^2+5s+3}{3(s+5)}}$$

$$V(s) = \frac{3(s+5)}{s(s^2+8s+18)}$$

$$I(s) = \frac{3(s+5)}{s(s^2+8s+18)} \cdot \frac{1}{s+5}$$

$$I(s) = \frac{3}{s(s^2+8s+18)} \text{ A}$$

$$V_o(s) + \frac{3 \cdot 5}{s(s^2+8s+18)} - \frac{3(s+5)}{s(s^2+8s+18)}$$

$$V_o(s) = \frac{3(s+5) - 15}{s(s^2+8s+18)}$$

$$V_o(s) = \frac{3s}{s(s^2+8s+18)}$$

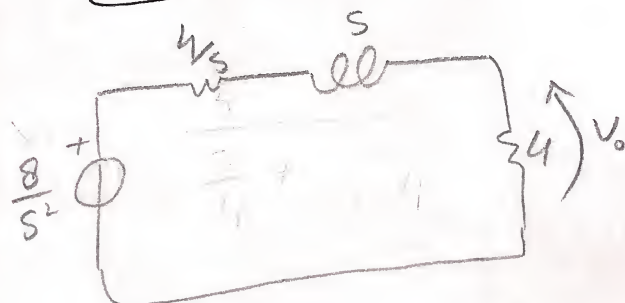
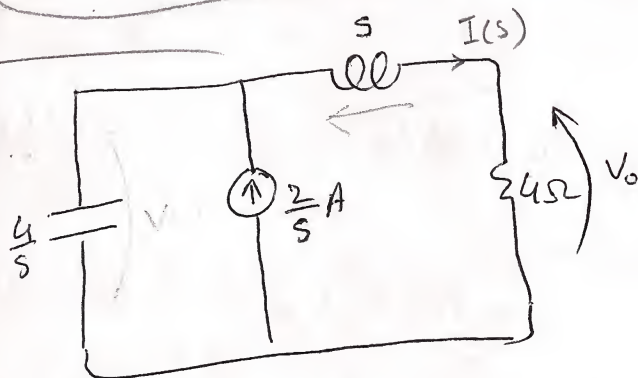
$$V_o(s) = \frac{3}{s^2+8s+18}$$

$$= \frac{3}{(s+4)^2 + (\sqrt{2})^2}$$

$$= \frac{3}{\sqrt{2}} \cdot \frac{\sqrt{2}}{(s+4)^2 + (\sqrt{2})^2}$$

$$V_o(t) = \frac{3}{\sqrt{2}} e^{-4t} \sin(\sqrt{2}t) u(t) \text{ V}$$

b)



$$V_o(s) = \frac{4}{4+s + \frac{4}{s}} \cdot \frac{8}{s^2}$$

$$V_o(s) = \frac{4}{4s+s^2+4} \cdot \frac{8}{s^2} = \frac{32}{s(s+2)^2}$$

$$s) = \frac{32}{s(s+2)^2}$$

$$\frac{32}{s(s+2)^2} = \frac{A}{s} + \frac{B}{(s+2)^2} + \frac{C}{s+2}$$

$$32 = A(s+2)^2 + B(s) + C(s+2)s$$

$$-s=0$$

$$32 = 4A \Rightarrow \boxed{A=8}$$

$$-s=-2$$

$$32 = -2B \Rightarrow \boxed{B=-16}$$

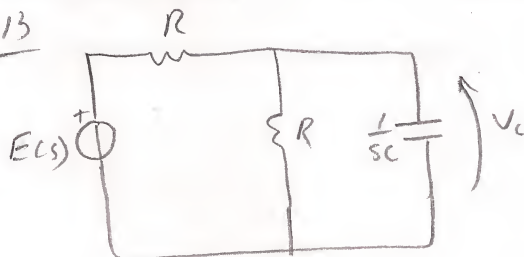
$$C = \lim_{s \rightarrow -2} \frac{d}{ds} \left((s+2)^2 \frac{32}{s(s+2)^2} \right)$$

$$C = \lim_{s \rightarrow -2} -32s^{-2} \Rightarrow \boxed{C=-8}$$

$$V_o(s) = \frac{8}{s} - \frac{16}{(s+2)^2} + \frac{-8}{s+2}$$

$$\boxed{V_o(t) = (8 - 16te^{-2t} - 8e^{-2t})u(t) \text{ V}}$$

11.13



$$e(t) = E_0 u(t) - E_0 u(t-T)$$

$$E(s) = \frac{E_0}{s} - E_0 \frac{e^{-sT}}{s}$$

$$V_c(s) = \frac{\frac{E(s)}{R}}{\frac{1}{R} + \frac{1}{R} + sC} = \frac{\frac{E(s)}{R}}{\frac{2+RSC}{R}}$$

$$V_c(s) = \frac{E(s)}{2+RSC}$$

$$V_c(s) = \frac{E_0(1-e^{-sT})}{s(2+RSC)} \Rightarrow V_c(s) = \frac{E_0(1-e^{-sT})}{s(2+RSC)}$$

$$V_c(s) = \frac{E_0}{RCS(s+\frac{2}{RC})} (1-e^{-sT})$$

$$\frac{E_0}{s(s+\frac{2}{RC})} = \frac{A}{s} + \frac{B}{s+\frac{2}{RC}}$$

$$-s=0$$

$$A = \frac{E_0}{s+\frac{2}{RC}} \Big|_0 = \frac{RCE_0}{2} \checkmark$$

$$-s = -\frac{2}{RC}$$

$$B = \frac{E_0}{s} \Big|_{-\frac{2}{RC}} = \frac{E_0 RC}{-2} \checkmark$$

$$\frac{E_0}{RCS(s+\frac{2}{RC})} = \frac{RCE_0}{RCE} \left(\frac{\frac{E_0}{2}}{s} - \frac{\frac{E_0}{2}}{s+\frac{2}{RC}} \right)$$

$$V_c(s) = \left[\frac{E_0}{2s} - \frac{E_0}{2(s+\frac{2}{RC})} \right] (1-e^{-sT})$$

$$V_c(t) = \left(\frac{E_0}{2} u(t) - \frac{E_0}{2} e^{-\frac{2}{RC}t} u(t) \right) (1-e^{-sT})$$

$$\boxed{V_c(t) = \left[\frac{E_0}{2} - \frac{E_0}{2} e^{-\frac{2}{RC}t} \right] u(t) - \left[\frac{E_0}{2} - \frac{E_0}{2} e^{-\frac{2}{RC}(t-T)} \right] u(t-T)}$$

Rectangular Form: $\hat{x} = a + jb$

$$\text{Magnitude} = \sqrt{a^2 + b^2} = |\hat{x}|$$

$$\text{Phase Angle} = \angle \hat{x} = \tan^{-1} \frac{b}{a}$$

if $\hat{x} = a + jb$, $\tan^{-1} \frac{b}{a} = \theta^\circ$

$$\hat{x} = a - jb, \quad \tan^{-1} \frac{-b}{a} = \theta^\circ$$

$$\hat{x} = -a + jb, \quad \left(\tan^{-1} \frac{b}{-a} \right) + 180^\circ = \theta^\circ$$

$$\hat{x} = -a - jb, \quad \left(\tan^{-1} \frac{b}{-a} \right) - 180^\circ = \theta^\circ$$

Phasor Form $\hat{x} = |\hat{x}| \angle \theta^\circ$

$$\hat{x} = |\hat{x}| e^{j\theta} \quad (\text{Polar Form})$$

$$\text{Sinusoidal Form} = |\hat{x}| \cos(\omega t + \theta^\circ) = \hat{x} \quad (i)$$

$$\omega = \text{Angular Frequency} = 2\pi f$$

$$\angle = \theta^\circ$$

$$\hat{x} = |\hat{x}| \sin(\omega t + \theta^\circ) \quad (ii)$$

$$\angle = \theta^\circ - 90^\circ$$

12.1

$$a) (2+j3)(-4-j5) = 7-j22$$

$$= -8 - 10j - 12j - j^2 15$$

$$= -8 + 15 - 22j = \boxed{7-j22} \checkmark$$

$$2+j3$$

$$-4-j5$$

$$r_1 = \sqrt{13}$$

$$r_2 = \sqrt{41}$$

$$\theta_1 = 56.309^\circ$$

$$\theta_2 = 51+180$$

$$\theta_2 = 231.34$$

$$(2+j3)(-4-j5) = \sqrt{13} \angle 56.309^\circ \cdot \sqrt{41} \angle 231.34^\circ$$

$$= 23.086 e^{j287.649}$$

$$= 23.086 (\cos(287.64) + j \sin(287.64))$$

$$= 7 - j22 \checkmark$$

$$b) \frac{-4-j5}{2+j3} = -\frac{23}{13} + j \frac{2}{13}$$

$$a) (2+j3)(-4-j5) = 7-j22$$

$$1) -8 - 10j - 12j - j^2 15$$

$$= -8 - 22j + 15$$

$$= 7 - 22j = R.H.S$$

$$2) 2+j3$$

$$r_1 = \sqrt{4+9}$$

$$r_1 = \sqrt{13}$$

$$\theta_1 = \tan^{-1} \frac{3}{2}$$

$$\theta_1 = 56.309$$

$$\frac{x+yj}{-4-j5}$$

$$-4-j5$$

$$r_2 = \sqrt{16+25}$$

$$r_2 = \sqrt{41}$$

$$\theta_2 = \tan^{-1} \frac{-5}{-4} + 180$$

$$\theta_2 = 231.34$$

$$(2+j3)(-4-j5) = \sqrt{13} \cdot \sqrt{41} e^{j(56.309 + 231.34)}$$

$$= 23.086 e^{j287.649}$$

$$= 23.086 (\cos(287.649) + j \sin(287.649))$$

$$= 6.99 - j21.99 = R.H.S$$

$$b) \frac{-4-j5}{2+j3} = -\frac{23}{13} + j \frac{2}{13}$$

$$(-4-j5)$$

$$2+j3$$

$$r_1 = \sqrt{16+25}$$

$$r_2 = \sqrt{4+9}$$

$$r_1 = \sqrt{41}$$

$$r_2 = \sqrt{13}$$

$$\theta_1 = 231.34$$

$$\theta_2 = 56.309$$

$$\frac{-4-j5}{2+j3} = \frac{\sqrt{41} e^{j231.34}}{\sqrt{13} e^{j56.309}}$$

$$= 1.77 e^{j175.031}$$

$$= 1.77 (\cos(175.031) + j \sin(175.031))$$

$$= -1.7633 + j0.1533 =$$

$$= -\frac{23}{13} + j \frac{2}{13} \checkmark$$

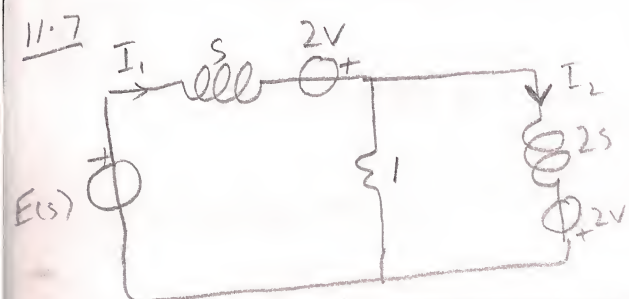
$$= \frac{5/4}{s} + \frac{-\frac{5}{4}s - \frac{25}{4}}{(s+2)^2 + (2)^2}$$

$$i(s) = \frac{5/4}{s} + \frac{-\frac{5s-20}{4}}{(s+2)^2 + (2)^2}$$

$$= \frac{5/4}{s} + \frac{-5/4}{(s+2)^2 + (2)^2}$$

$$= \frac{5}{4} \left[\frac{1}{s} - \left(\frac{s+2}{(s+2)^2 + (2)^2} + \frac{2}{(s+2)^2 + (2)^2} \right) \right]$$

$$= \frac{5}{4} \left(u(t) - (e^{-2t} \cos(2t) + e^{-2t} \sin(2t)) u(t) \right)$$



12.2

a) $v(t) = 20 \cos(150t - 60^\circ) \text{ V}$

$$V(t) = 20 e^{-60j}$$

$$= 20 (\cos(-60) + j \sin(-60))$$

$$= 10 - j \frac{\sqrt{3}}{2} \cdot 10$$

$$\hat{V} = \frac{10}{\sqrt{2}} - j \frac{\sqrt{3}}{2} 10$$

b) $v(t) = 10 \cos(1000t + 180^\circ) \text{ V}$

$$v(t) = 10 e^{180j}$$

$$= 10 (\cos(180) + j \sin(180))$$

$$\hat{V} = -10$$

$$\hat{V} = -\frac{10}{\sqrt{2}} \text{ V}$$

c) $i(t) = -4 \cos(3t) + 3 \cos(3t - 90^\circ)$

$$i(t) = -4 e^{0j} + 3 e^{-90j}$$

$$= -4 (\cos(0) + j \sin(0)) + 3 (\cos(-90) + j \sin(-90))$$

$$i(t) = -4 - 3j$$

$$\hat{I} = \frac{i(t)}{\sqrt{2}} = \frac{-4}{\sqrt{2}} - \frac{3}{\sqrt{2}} j \text{ A}$$

12.3 a) $\hat{V} = (169 \angle -45^\circ) \text{ V}$

$$f = 60 \text{ Hz} \quad \omega = 2\pi f = 120\pi$$

$$\frac{v(t)}{\sqrt{2}} = 169 e^{-45j}$$

$$= 169 (\cos(-45) + j \sin(-45))$$

$$\frac{v(t)}{\sqrt{2}} = \frac{169}{\sqrt{2}} - j \frac{169}{\sqrt{2}}$$

$$v(t) = 169 - j 169$$

$$\frac{v(t)}{\sqrt{2}} = 169 \cos(120\pi t - 45^\circ)$$

$$v(t) = 169 \sqrt{2} \cos(120\pi t - 45^\circ) \text{ V}$$

$$\omega = 10 \text{ krad/s}$$

b) $\hat{V} = (10 \angle 90^\circ + 66 - j10) \text{ V}$

$$\frac{v(t)}{\sqrt{2}} = 10 e^{90j} + 66 - j10$$

$$= 10 (\cos(90) + j \sin(90)) + 66 - j10$$

$$= j10 + 66 - j10 = 66$$

$$\frac{v(t)}{\sqrt{2}} = 66$$

$$\phi = 0$$

$$r = 66$$

$$v(t) = 66 \sqrt{2} \cos(10^4 t) \text{ V}$$

$$c) \hat{I} = (15 + j5 + 10 \angle 180^\circ) \text{ mA} \quad \omega = 1 \text{ k rad/s}$$

$$\frac{i(t)}{\sqrt{2}} = 15 + j5 + 10(\cos 180^\circ + j \sin 180^\circ) \text{ mA}$$

$$= 15 + j5 - 10$$

$$\frac{i(t)}{\sqrt{2}} = 5 + j5 \quad r = \sqrt{50} = 5\sqrt{2}$$

$$\frac{i(t)}{\sqrt{2}} = 5\sqrt{2} \cos(10^3 t + 45^\circ) \quad \theta = \tan^{-1}(1) = 45^\circ$$

$$i(t) = 10 \cos(10^3 t + 45^\circ) \text{ mA}$$

$$12.4 \quad \omega = 200 \text{ rad/s}$$

$$a) \hat{V}_1 = \frac{10 + j10}{2 - j3} \times \frac{2 + j3}{2 + j3}$$

$$= \frac{20 + 30j + 20j - 30}{4 + 9}$$

$$\hat{V}_1 = \frac{-10 + 50j}{13}$$

$$r = \sqrt{\frac{100 + 2500}{169}} = \frac{\sqrt{2600}}{13} = \frac{10\sqrt{26}}{13}$$

$$\phi = \tan^{-1} \frac{50}{-10} + 180^\circ = 101.309^\circ$$

$$\frac{v_1(t)}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{13}} \cos(200t + 101.309^\circ)$$

$$v_1(t) = \frac{20}{\sqrt{13}} \cos(200t + 101.31^\circ)$$

$$b) \hat{V}_2 = (3 - 8j)(5e^{j60^\circ}) \text{ V}$$

$$= (3 - 8j)(5(\cos(-60^\circ) - j \sin(60^\circ)))$$

$$= (3 - 8j)\left(\frac{5}{2} - \frac{5\sqrt{3}}{2}j\right)$$

$$= \frac{15}{2} - \frac{15\sqrt{3}}{2}j - 20j - 20\sqrt{3}$$

$$= \frac{15 - 40\sqrt{3}}{2} - j \frac{15\sqrt{3} + 40}{2}$$

-27.141 32.99038

$$r = 5\sqrt{13}$$

$$\phi = 50.55 + 180 =$$

$$v_2(t) = 5\sqrt{146} \cos(200t + 50.55^\circ)$$

$$\frac{v_2(t)}{\sqrt{2}} = \sqrt{13} e^{-69.44^\circ} 5 e^{-j60^\circ}$$

$$= 5\sqrt{146} e^{-129.44^\circ}$$

$$v_2(t) = 5\sqrt{146} \cos(200t - 129.44^\circ) \text{ V}$$

$$c) \hat{I}_1 = \frac{10}{1 + j3} \times \frac{1 - j3}{1 - j3}$$

$$= \frac{10 - 30j}{1 + 9} = 1 - 3j$$

$$\frac{i_1(t)}{\sqrt{2}} = \sqrt{10} e^{-71.56^\circ}$$

$$i_1(t) = \sqrt{20} \cos(200t - 71.56^\circ)$$

$$\frac{1}{Z} = \frac{1+3j}{1-j3} A$$

$$\frac{i_2(t)}{\sqrt{2}} = \frac{\sqrt{10} e^{71.56j}}{\sqrt{10} e^{-71.56j}} = 1 e^{143.13j}$$

$$i_2(t) = \sqrt{2} \cos(200t + 143.13^\circ) A$$

12.5

$$v_1(t) = 50 \cos(\omega t - 45^\circ)$$

$$v_2(t) = 25 \sin(\omega t)$$

$$v_3(t) = -[v_1(t) + v_2(t)]$$

$$= -50 e^{-45j} - 25 e^{-90j}$$

$$= 50(\cos(-45^\circ) + j \sin(-45^\circ)) - 25(\cos(90^\circ) + j \sin(90^\circ))$$

$$= -\frac{50}{\sqrt{2}} + \frac{50}{\sqrt{2}} j + 25 j$$

$$= -35.35 + 60.355j$$

$$r = 69.948$$

$$\phi = \tan^{-1} \frac{60.355}{-35.35} + 180$$

$$\phi = 120.36^\circ$$

$$v_3(t) = 69.948 \cos(\omega t + 120.36^\circ) V$$

$$12.6 \quad \hat{V}_1 = 5 e^{j0}, \quad \hat{V}_2 = 10 e^{-135j}$$

$$\frac{v_1(t)}{\sqrt{2}} = 5(\cos(0^\circ) + j \sin(0^\circ)) \quad \frac{v_2(t)}{\sqrt{2}} = 10(\cos(135^\circ) - j \sin(135^\circ))$$

$$v_1(t) = 5\sqrt{2} V$$

$$r = 5\sqrt{2}$$

$$\phi = 0$$

$$v_1(t) = 5\sqrt{2} \cos(\omega t)$$

$$v_2(t) = 10\sqrt{2} \cos(\omega t - 135^\circ)$$

$$r = 10\sqrt{2}$$

$$\phi = 45 - 180 = -135^\circ$$

$$v_2(t) = 10\sqrt{2} \cos(\omega t - 135^\circ)$$

$$v_3(t) = 5\sqrt{2} - 10 - 10j$$

$$v_3(t) = -2.928 - 10j$$

$$v_3(t) = 10.42$$

$$\phi = 253.67^\circ$$

$$v_3(t) = 10.42 \cos(\omega t + 253.67^\circ)$$

12.7

$$\hat{V}_1 = (2 + 6j) V$$

$$\frac{v_1(t)}{\sqrt{2}} = \sqrt{40} e^{71.56j}$$

$$\hat{V}_1 = 6.32 e^{71.56j}$$

$$\hat{V}_1 = 6.32 e^{11.565j}$$

$$\hat{V}_1 = 6.32 \cos(\omega t + 11.565^\circ) V$$

$$\hat{V}_1 = 6.32 (\cos(11.565^\circ) + j \sin(11.565^\circ))$$

$$= 6.32 (0.979 + j 0.2004)$$

$$\hat{V}_1 = 6.18718 + 1.26j$$

$$12.8 \quad \hat{V}_1 = -3 + j4$$

$$V_{2m} = 10V$$

$$v_1(t) = -4.2426 + 5.656j$$

$$\phi_1 = -53.13 + 180 = 126.86^\circ$$

$$\phi_2 = 216.86^\circ$$

$$v_2(t) = 10 \cos(\omega t + 216.86^\circ)$$



$$Z_{in} = 50 + j40 \Omega$$

b)

$$\frac{1}{j\omega C} = \frac{-j}{\omega C} = \frac{-j}{200 \cdot 200 \cdot 10^3 \cdot 10^9}$$

$$= \frac{-j}{4 \times 10^2} = \frac{-j}{0.04}$$

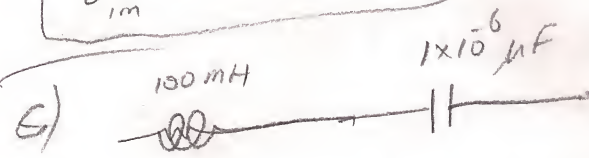
$$Z_{in} = \frac{-50j}{0.04} = \frac{-5000j}{4} = \frac{-5000j}{200 - j100}$$

$$Z_{in} = \frac{-5000j}{200 - j100} \times \frac{200 + j100}{200 + j100}$$

$$= \frac{-50j}{2 - j} \times \frac{2 + j}{2 + j}$$

$$= \frac{-100j - 50j^2}{4 + 1} = \frac{50 - 100j}{5}$$

$$Z_{in} = 10 - 20j$$



$$j\omega L = j2 \times 10^3 \cdot 100 \times 10^{-3} = 200j$$

$$\frac{-j}{\omega C} = \frac{-j}{2 \times 10^3 \times 10^{-6}} = \frac{-j10^3}{2}$$

$$Z_{in} = 200j - \frac{j10^3}{2} = j\left(\frac{400 - 1000}{2}\right)$$

$$Z_{in} = -300j$$



$$Z_{in} = 10 - \frac{j}{\omega 0.01 \times 10^{-6}}$$

$$\phi = 12.5^\circ$$

$$\phi = \tan^{-1}\left(\frac{-1}{\omega 0.01 \times 10^{-6}}\right)$$

$$\tan 12.5^\circ = \frac{-1}{\omega (0.01 \times 10^{-6})}$$

$$\omega = -4.5 \times 10^7 \text{ rad/s}$$

$$f = \frac{\omega}{2\pi}$$

$$f = -7.18 \times 10^6 \text{ Hz}$$

$$\hat{V}_1 = 20 \angle 10^\circ \text{ V} \quad \hat{V}_2 = (9 - j17) \text{ V}$$

$$r_1 = 20$$

$$r_2 = 19.23$$

a) \hat{V}_1 has greater mag.

b) $v_1(t) - v_2(t)$

$$\frac{v_1(t)}{\sqrt{2}} = 20(\cos 10 + j \sin 10)$$

$$= 19.69 + j3.47$$

$$v_1(t) = 27.84 + j4.911$$

$$v_2(t) = 9 - j17 = 12.72 - j24.041$$

$$v_1(t) - v_2(t) = 15.134 + j28.952j$$

$$\angle v_3(t) = 62.40 \quad v_{3m} = 32.67$$

$$v_3(t) = 32.67 \cos(\omega t + 62.40)$$

$$v_3(0) = 15.13 \text{ V}$$

$$v_3(t) = -9 + j17 + 27.84 + j4.911$$

$$v_3(t) = -18.845 - j21.911$$

$$= 18.845 \angle 21.911$$

$$V_{3m} = 28.8902V$$

$$\phi_3 = 49.302^\circ$$

$$V_3(t) = 28.8902 \cos(49.302^\circ)$$

$$V_3(0) = 19.497$$

$$\hat{V}_2 = 9 - j17$$

$$\hat{V}_1 = 19.98 + 3.47j$$

$$\hat{V}_3 = \hat{V}_1 - V_2$$

$$= 10.98 + 20.47j$$

$$V_3(t) = 15.528 + 28.94j$$

$$V_{3m} = 32.8427$$

$$\phi_3 = 61.7838^\circ$$

$$V_3(t) = 32.8427 \cos(\omega t + 61.7838^\circ)$$

$$V_3(0) = 15.52V$$

$$V_1(t) = 28.255 + 4.90j$$

$$r = 28.677 \quad \phi = 9.838^\circ$$

$$V_1(t) = 28.677 \cos(\omega t + 9.838^\circ)$$

$$\cos \omega t \cos 9.838^\circ - \sin \omega t \sin 9.838^\circ$$

$$\cos(314t + 9.838^\circ) = 0$$

$$V_1(t) = 27.84 + j4.911$$

$$r = 28.26 \quad \phi = 10^\circ$$

$$\phi = 28.26 (\cos 100\pi t + 10^\circ)$$

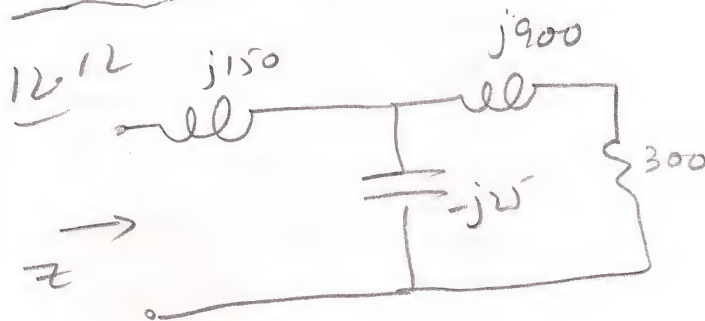
$$100\pi t = 80$$

$$t = \frac{80}{100\pi}$$

$$V_1 = 20\sqrt{2} \cos(314t + 10^\circ)$$

$$314t = 90 - 10$$

$$t = \frac{80}{314} = 0.25$$



$$Z = \frac{(300 + j900) \cdot -j25}{(300 + j900) - j25} + j150$$

$$Z = \frac{-7500j + 22500}{300 + 875j} + j150$$

$$Z = \frac{-7500j + 22500 + 45000j - 131250}{300 + 875j}$$

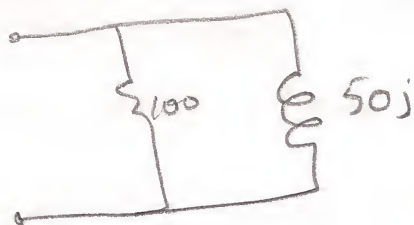
$$Z = \frac{37500j - 108750}{300 + 875j} \times \frac{300 - 875j}{300 - 875j}$$

$$= \frac{11250000j - 32625000 + 32812500 + 90000 + 765625}{855625}$$

$$= \frac{106406250j + 187500}{855625}$$

$$Z = 0.22 + 124.36j$$

12.13

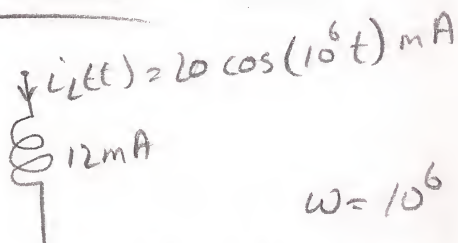


$$Z = \frac{5000j}{100 + 50j} \times \frac{100 - 50j}{100 - 50j}$$

$$Z = \frac{500000j + 250000}{10000 + 2500}$$

$$\boxed{Z = 40j + 20}$$

12.14



$$\omega = 10^6$$

$$Z = j\omega L = j10^6 12 \text{mH} = j12 \text{k}\Omega$$

$$V_L = i_L(t) Z$$

$$= j12 \times 10^3 \cdot 20 \cos(10^6 t) \cdot 10^{-3}$$

$$\hat{V} = j \frac{240}{\sqrt{2}} e^{j(t)}$$

$$V(t) = L \frac{d}{dt} 20 \cos(10^6 t) \text{m}$$

$$= 12 \cdot 10^3 \cdot 20 (-\sin(10^6 t)) \cdot 10^{-3}$$

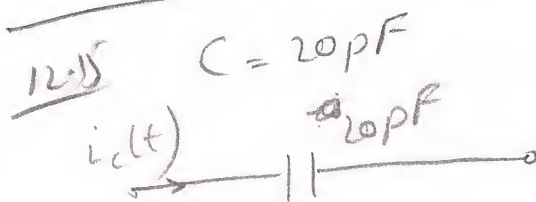
$$= -240 e^{-90j}$$

$$= -240 (\cos(-90) + j \sin(-90))$$

$$V(t) = 240j$$

$$\boxed{\hat{V} = \frac{240}{\sqrt{2}} j}$$

$$c) \boxed{V_L(t) = -240 \sin(10^6 t) \text{V}}$$



$$a) Z_C = \frac{-j}{\omega C} = \frac{-j}{10^6 \cdot 20 \cdot 10^{-12}} = -j \frac{10^6}{20}$$

$$\boxed{Z = -j50 \text{k}\Omega}$$

$$b) \cancel{V_L(t) = f}$$

$$\cancel{V_L(t) = -j50 \text{k} \cdot 0.3 \text{m} \cos(10^6 t)}$$

$$i_C(t) = C \frac{dV(t)}{dt}$$

$$V(t) = \frac{1}{C} \int i_C(t) dt$$

$$= \frac{10^{-12}}{20} \int 0.3 \cos(10^6 t) 10^{-3} dt$$

$$= \frac{10^{-3} \cdot 0.3 \times 10^6}{20} \int 10^6 \cos(10^6 t) dt$$

$$= \frac{10^3 \cdot 0.3}{20} \sin(10^6 t)$$

$$= 15 \sin(10^6 t) \checkmark (c)$$

$$= 15 e^{-90j}$$

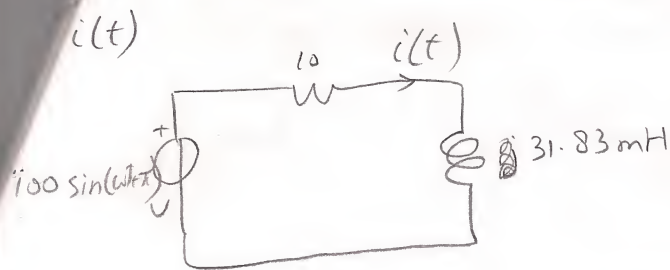
$$= 15 (\cos(-90) - j \sin 90)$$

$$V(t) = -15j$$

$$\boxed{\hat{V} = \frac{-15}{\sqrt{2}} j}$$

$$f = 50 \text{ Hz} \quad e(t) = 100 \sin(\omega t + \pi) \text{ V}$$

$$v(t) = 0.8 \cos(1000t - 20^\circ) (62.883 + 33.218j)$$



$$i(t) = \frac{(j31.83 \text{ m} + 10) / 100 \sin(\omega t + \pi)}{}$$

$$= \frac{10 \angle 72.55^\circ / 100 \angle 180 - 90^\circ}{}$$

$$= 1000 \angle 11.55^\circ$$

$$= 1000 \angle 162.558^\circ$$

$$i(t) =$$

$$j\omega L = 100\pi \times 31.83 \text{ m} j = j9994.62 \times 10^{-3} = 9.994 j 3.183 \pi$$

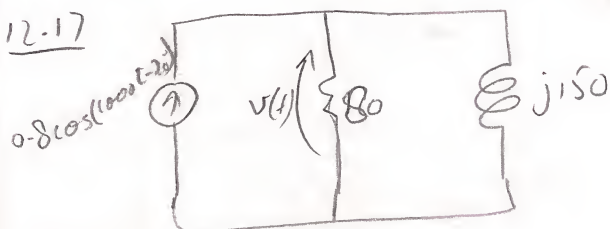
$$i(t) = \frac{100 \sin(\omega t + \pi)}{10 + 9.994 j 3.183 \pi}$$

$$= \frac{100 e^{j\pi}}{14.13 e^{j44.99^\circ}}$$

$$= 7.077 e^{j45^\circ}$$

$$i(t) = 7.077 \cos(\omega t + 45^\circ) \text{ A}$$

12.17



$$Z_{eq} = \frac{12000 j}{80 + 150 j} \times \frac{80 - 150 j}{80 - 150 j}$$

$$Z_{eq} = \frac{960000 j + 1800000}{28900}$$

$$Z_{eq} = 62.283 + 33.218 j$$

$$= 0.8 e^{-20j} \cdot 70.587 e^{28.073j} = 56.469 e^{8.072j}$$

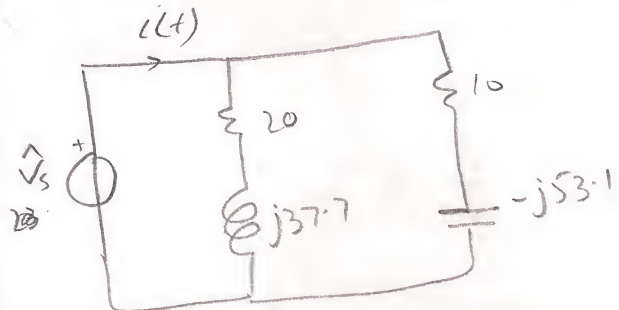
$$v(t) = 56.469 \cos(1000t + 8.072^\circ) \text{ V}$$

12.18 $i(t) = ?$

$$f = 60 \text{ Hz} \quad \hat{V}_s = 230 \angle 0^\circ \text{ V}$$

$$R_1 = 20 \quad R_2 = 10 \quad Z_L = j37.7$$

$$Z_C = -j53.1$$



$$V_s(t) = 230 \cos(120\pi t)$$

$$V_s(t) = 325.26 \cos(376.8t)$$

$$Z_{eq} = \frac{(10 - j53.1)(20 + j37.7)}{10 - j53.1 + 20 + j37.7}$$

$$Z_{eq} = \frac{200 + 377j - 1062j + 2001.87}{30 - j15.4}$$

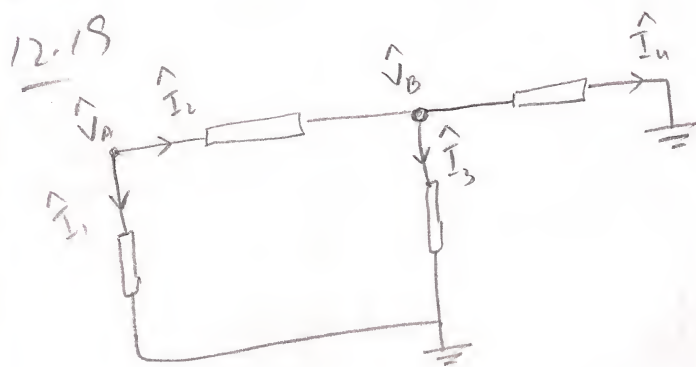
$$Z_{eq} = \frac{2201.87 - 685j}{30 - 15.4j} \times \frac{30 + 15.4j}{30 + 15.4j} = \frac{66056.1 - 20550j + 33908.79j + 10549}{1137.16}$$

$$= \frac{76605.1 + 13358.79j}{1137.16}$$

$$Z_{eq} = 67.365 + 11.747j$$

$$i(t) = 325.26 \angle 0^\circ / (67.365 + 11.747j) \\ = 325.26 \angle 0^\circ / 68.38 \angle 9.89^\circ \\ = 2224.27 \angle -9.89^\circ \\ = 4.75 \angle -9.89^\circ$$

$$i(t) = 4.75 \cos(376.8t - 9.89^\circ) \text{ A}$$



$$V_A(t) = 10 \cos(\omega t) \text{ V}$$

$$V_A(t) = 10 \angle 0^\circ$$

$$\hat{V}_A = \frac{10}{\sqrt{2}} \text{ V}$$

$$V_B(t) = 10 \sin(\omega t)$$

$$= 10 e^{-90j}$$

$$= 10 (\cos(-90) - j \sin(90))$$

$$\hat{V}_B = -\frac{10}{\sqrt{2}} j$$

$$i(t) = 5 \cos(\omega t + 135^\circ) \text{ A}$$

$$\hat{I}_1 = \frac{5}{\sqrt{2}} e^{135j}$$

$$= \cos 135 + j \sin 135$$

$$\hat{I}_1 = \left(-\frac{1}{\sqrt{2}} + j \frac{1}{\sqrt{2}}\right)$$

$$i_u(t) = \cos(\omega t)$$

$$\hat{I}_u = \frac{1}{\sqrt{2}}$$

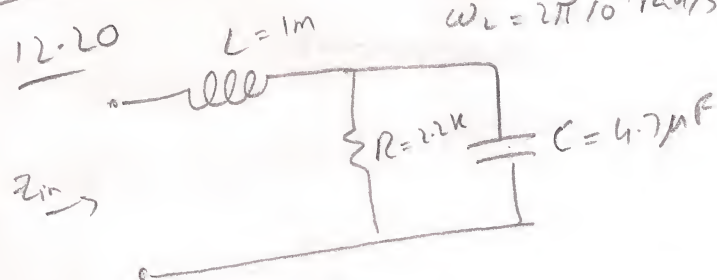
$$I_2 = -\hat{I}_1$$

$$\hat{I}_3 = \hat{I}_2 - \hat{I}_u$$

$$= \frac{1}{\sqrt{2}} - j \frac{1}{\sqrt{2}} - \frac{1}{\sqrt{2}}$$

$$\hat{I}_3 = -j \frac{1}{\sqrt{2}} \text{ A}$$

12.20



$$\omega_1 = 2\pi 10 \text{ rad/s}$$

$$\omega_2 = 2\pi 10^4 \text{ rad/s}$$

$$\frac{-j}{\omega_1 C} = \frac{-j}{2\pi 10 \times 4.7 \times 10^{-6}} = \frac{-j}{0.000295}$$

$$\frac{-j}{\omega_2 C} = \frac{-j}{2\pi 10^4 \times 4.7 \times 10^{-6}} = \frac{-j}{0.29516}$$

$$j\omega_1 L = j 2\pi 10 \times 10^{-3} = j 0.0628$$

$$j\omega_2 L = j 2\pi 10^4 \times 10^{-3} = j 62.8$$

$$Z_{in}(\omega_1) = \frac{2.2 \times 10^3 \left(\frac{-j}{0.000295}\right) + j 0.0628}{2200 + \frac{j}{0.000295}}$$

$$= \frac{-7457627.119j}{2200 - 3389.83j} + j 0.0628$$

$$= \frac{-7457627.119j + j 3.816 + 212.8813}{2200 - 3389.83j}$$

$$= \frac{-7457613.303j + 212.8813}{2200 - 3389.83j}$$

$$V_1 = \frac{-7457613.303j + 212.8813}{2200 - 3389.83j} \times \frac{2200 + 3389.83j}{2200 + 3389.83j}$$

$$= \frac{-1.64067 \times 10^{10}j + 2.528 \times 10^{18} + 468338.86 + 721631.41j}{16330947.43}$$

$$Z_{in}(\omega) = 1.5 \times 10^{11} - j 1.01 \times 10^3$$

$$Z_{in}(\omega) = 62.8j + \frac{-2200j}{0.29516} \times \frac{2200 - j}{0.29516}$$

$$= 62.8j + \frac{-2200j}{649.352 - j}$$

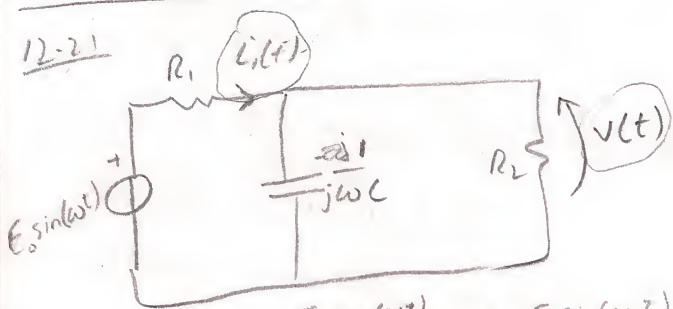
$$= \frac{40779.305j + 62.8 - 2200j}{649.352 - j}$$

$$= \frac{62.8 + 38579.305j}{649.352 - j} \times \frac{649.352 + j}{649.352 + j}$$

$$= \frac{25051549.25j - 38579.3056 + 40779.3056 + 62.8j}{421659.0199}$$

$$= \frac{2200 + 25051612.05j}{421659.0199}$$

$$Z_{in}(\omega) = 0.0052 + 59.41j$$



$$V(t) = \frac{E_0 \sin(\omega t)}{\frac{1}{R_1} + j\omega C + \frac{1}{R_2}} = \frac{E_0 \sin(\omega t)}{\frac{R_2 + R_1 R_2 j\omega C + R_1}{R_1 R_2}}$$

$$V(t) = \frac{R_2}{R_1 + R_2 + j R_1 R_2 \omega C} E_0 e^{-90j}$$

$$V(t) = \frac{E_0 R_2}{R_1 + R_2 + j R_1 R_2 \omega C} \cdot -j$$

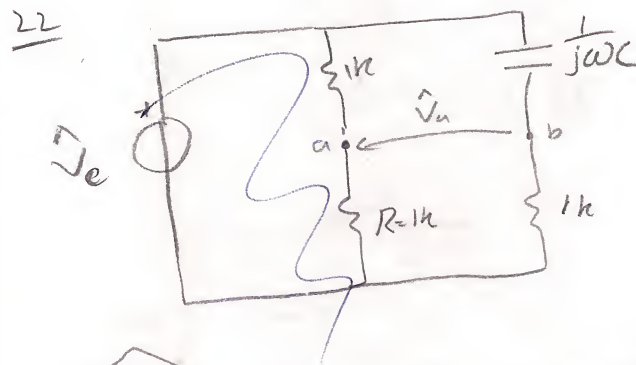
$$V(t) = \frac{-E_0 R_2 j}{(R_1 + R_2) + j(R_1 R_2 \omega C)} \times \frac{(R_1 + R_2) - j(R_1 R_2 \omega C)}{(R_1 + R_2) - j(R_1 R_2 \omega C)}$$

$$V(t) = \frac{-E_0 R_2 (R_1 + R_2) j - E_0 R_2^2 R_1 \omega C}{(R_1 + R_2)^2 + (R_1 R_2 \omega C)^2}$$

$$|V(t)| = \sqrt{\frac{(E_0 R_1 R_2^2 \omega C)^2}{((R_1 + R_2)^2 + (R_1 R_2 \omega C)^2)^2} + \frac{(E_0 R_2 (R_1 + R_2))^2}{((R_1 + R_2)^2 + (R_1 R_2 \omega C)^2)^2}}$$

$$|V(t)| = \frac{E_0 R_2 \sqrt{(R_1 + R_2)^2 + (R_1 R_2 \omega C)^2}}{(R_1 + R_2)^2 + (R_1 R_2 \omega C)^2}$$

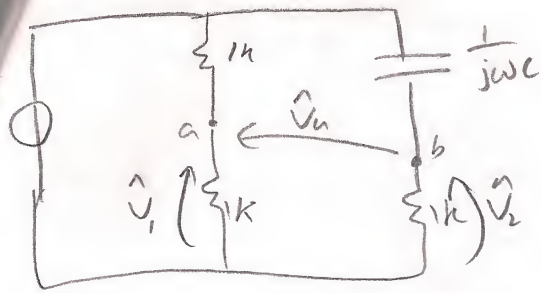
$$|V(t)| = \frac{E_0 R_2}{\sqrt{(R_1 + R_2)^2 + (R_1 R_2 \omega C)^2}}$$



$$\frac{Z_2}{Z_1 + Z_2} = \frac{Z_2}{Z_1 + Z_2}$$

$$\arctan \frac{E_0 R_2 (R_1 + R_2)}{E_0 R_2^2 R_1 \omega C}$$

$$\arctan \left(\frac{R_1 + R_2}{\omega R_1 R_2 C} \right)$$



$$\hat{V}_1 = \frac{\hat{V}_e}{2}$$

$$\hat{V}_2 = \frac{R}{R + \frac{1}{j\omega C}} \hat{V}_e = \frac{j\omega RC}{j\omega RC + 1} \hat{V}_e$$

$$\begin{aligned} \hat{V}_u &= \hat{V}_1 - \hat{V}_2 \\ &= \hat{V}_e \left(\frac{1}{2} - \frac{j\omega RC}{j\omega RC + 1} \right) \\ &= \hat{V}_e \left(\frac{j\omega RC + 1 - 2j\omega RC}{2(j\omega RC + 1)} \right) \end{aligned}$$

$$\hat{V}_u = \left(\frac{1 - j\omega RC}{2(1 + j\omega RC)} \right) \hat{V}_e$$

$$2 \frac{\hat{V}_u}{\hat{V}_e} = \frac{1 - j\omega RC}{1 + j\omega RC}$$

since \hat{V}_u lags \hat{V}_e by 120°

$$\angle \hat{V}_u - \angle \hat{V}_e = -120$$

As $\angle 1 - jx = -\arctan x$

$\angle 1 + jn = \arctan n$

$\angle \frac{1 - jn}{1 + jn} = -2\arctan n$

$\angle \frac{1 - j\omega RC}{1 + j\omega RC} = -2\arctan x$

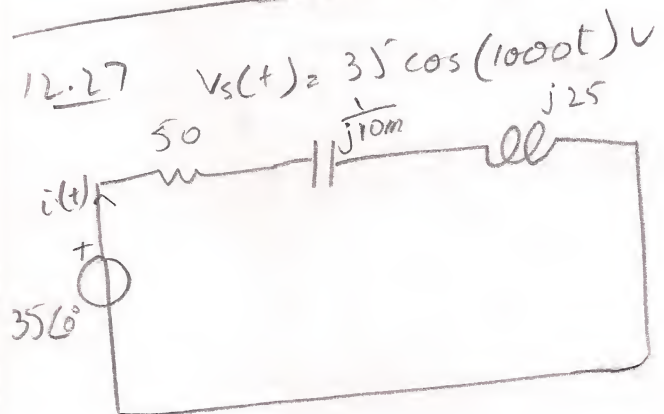
$$+ \angle \arctan \omega RC = +120^\circ$$

$$10^3 \times 62.8 \times 10^3 C = 1.73$$

$$C = \frac{1.73}{62.8} \times 10^{-5}$$

$$C = 27.5 \times 10^{-9} \text{ F}$$

$$C = 27.5 \text{ nF}$$



$$i(t) = \frac{35 \angle 0^\circ}{50 + \frac{1}{j10} + j25}$$

$$= \frac{35 \angle 0^\circ}{50 - 10j + 25j}$$

$$= \frac{35 \angle 0^\circ}{50 + 15j}$$

$$= \frac{35 \angle 0^\circ}{54.629 \angle 15.999^\circ} = 0.640 \angle -15.999^\circ$$

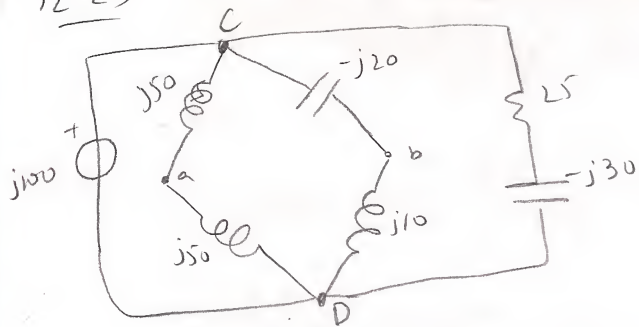
$$i(t) = 0.25 e$$

$$\hat{I} = \frac{i(t)}{\omega} = \frac{0.388}{\omega} \angle 56.3^\circ$$

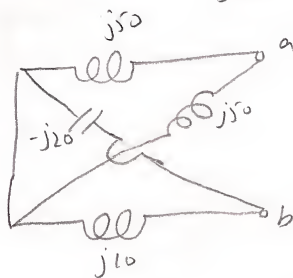
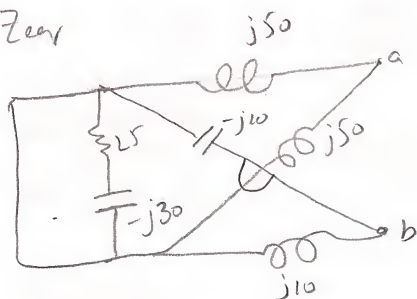
$$\hat{I} = 0.27 e^{j56.31^\circ}$$

E

12.23



$-Z_{eq}$

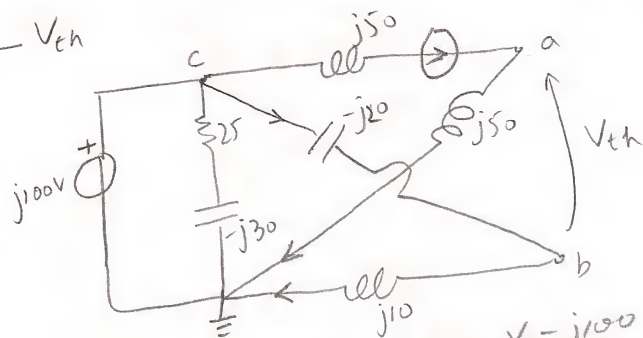


$$Z_{eq} = \frac{-j20 \times j10}{-j20 + j10} + \frac{j50 \times j50}{j50 + j50}$$

$$= \frac{200}{-j10} + \frac{-2500}{100j}$$

$$Z_{eq} = 20j + 25j = j45\Omega$$

$-V_{th}$



$$V_c = j100$$

$$-\frac{V_a - j100}{j50} = \frac{V_a}{j50}$$

$$V_a \left(-\frac{1}{j50} - \frac{1}{j50} \right) = -\frac{100}{50}$$

$$V_a \left(-\frac{2}{j50} \right) = -2 \Rightarrow V_a = +j50V$$

$$+\frac{V_b - j100}{+j20} = \frac{V_b}{j10}$$

$$V_b \left(\frac{1}{j20} - \frac{1}{j10} \right) = 50$$

$$V_b \left(\frac{1-2}{j20} \right) = 50$$

$$V_b = -100j$$

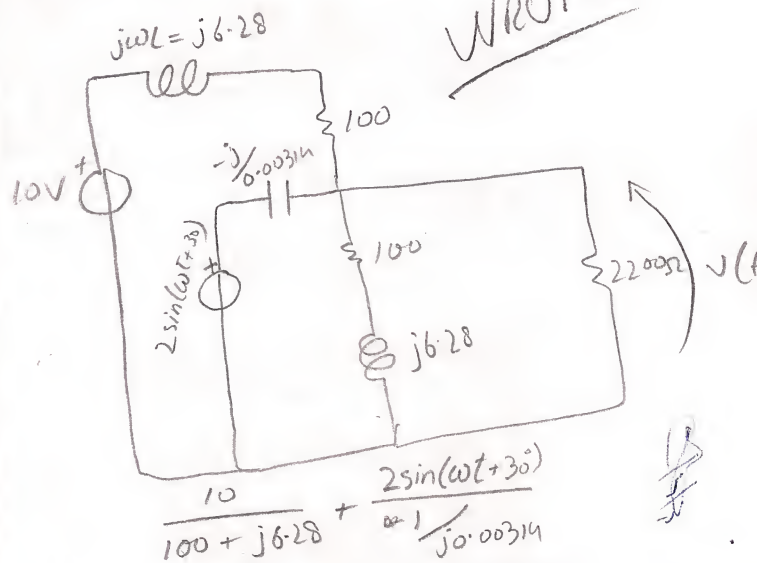
$$V_{th} = V_a - V_b$$

$$= +50j + 100j = 150j \text{ A}$$

12.24 $V(t)$? $E_0 = 10V$ $f = 1000Hz$

$e(t) = 2\sin(\omega t + 30^\circ)$ $L = 1mH$

$C = 0.5\mu F$ $R_1 = 100\Omega$ $R_2 = 2200\Omega$



$$V(t) = \frac{10}{100 + j6.28} + \frac{2\sin(\omega t + 30^\circ)}{1/j0.00314 + 100} \times \frac{2200}{2200 + 100 + j6.28}$$

$$2\sin(\omega t + 30^\circ) = 2e^{-60j} = 2(\cos(60) - j\sin(60))$$

$$= 1 - 1.732j$$

$$V(t) = \frac{\frac{10}{100 + j6.28} + \frac{1 - 1.732j}{j0.00314 + 100} \times \frac{2200}{2200 + 100 + j6.28}}{2200(100 + j6.28)}$$

$$R) = 0.27 e^{j56.31^\circ} \times 50$$

$$\hat{V}(R) = 13.728 e^{j56.31^\circ} \quad (12.27)$$

$$\hat{V}(-j10^2) = 0.27 e^{j56.31^\circ} (-j10^2)$$

$$= 0.27 (\cos 56.31^\circ + j \sin 56.31^\circ) (-j10^2)$$

$$= (0.149 + j0.2246) (-100j)$$

$$= -1.49j + 22.46$$

$$= 26.95 \angle -33.56^\circ$$

$$\hat{V}(C) = 26.95 e^{-33.56^\circ j}$$

$$\hat{V}(j25) = 0.27 e^{j56.31^\circ} (j25)$$

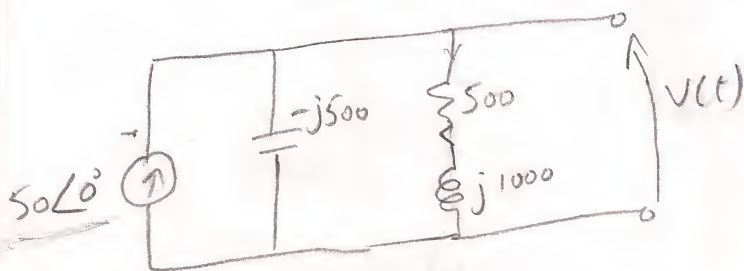
$$= (0.149 + j0.2246) (j25)$$

$$= 3.725j - 5.615$$

$$\hat{V}(j25) = 6.738 \angle 146.43^\circ$$

$$\hat{V}_L = 6.738 e^{146.43^\circ j}$$

12.28 $i_s(t) = 50 \cos(2000t) \text{ mA}$



$$Z_{eq} = \frac{(500 + j1000)(-j500)}{500 + j1000 - j500}$$

$$Z_{eq} = \frac{-j250000 + 500000}{500 + j500}$$

$$V(t) = \frac{500000 - j250000}{\frac{500}{10}(1+j)} m$$

$$V(t) = \frac{50000 - j25000}{1+j} \times \frac{1-j}{1-j} m$$

$$V(t) = \frac{50000 - 50000j - j25000 - 25000}{2} m$$

$$V(t) = \frac{25000 - 75000j}{2} m$$

$$V(t) = 12500 - 37500j m$$

$$V(t) = 39528.4707 \angle -71.565^\circ m$$

$$\hat{V} = 27950 e^{-71.565^\circ j} m \quad \checkmark$$

$$\hat{V} = 27.95 e^{-71.565^\circ j} V$$

c)

~~$i(t)$~~ $R=L$

$$i(t) = \frac{12.5(-37.5j)}{500 + j1000}$$

$$i(t) = \frac{(12.5 - 37.5j)(500 - j1000)}{250000 + 1000000}$$

$$= \frac{62500 - 12500j - 18750j - 37500}{1250000}$$

$$= \frac{-31250 - 31250j}{1250000}$$

$$i(t) = -0.025 - 0.025j$$

$$= 0.035 \angle -135^\circ$$

$$\hat{I} = 0.025 e^{-135^\circ j}$$

$$i(t) = 0.035 \cos(2000t - 135^\circ) A$$

$$i_c(t) = \frac{12.5 - 37.5j}{-j500}$$

$$i_c(t) = \frac{(12.5 - 37.5j) j500}{250000}$$

$$= \frac{6250 - 18750j}{250000}$$

$$i_c(t) = 0.025 - 0.075j$$

$$i_c(t) = \frac{0.07905}{0.00625} \angle -71.56$$

$$\hat{I}_c = \cancel{0.0044} 0.055 e^{-j71.56}$$

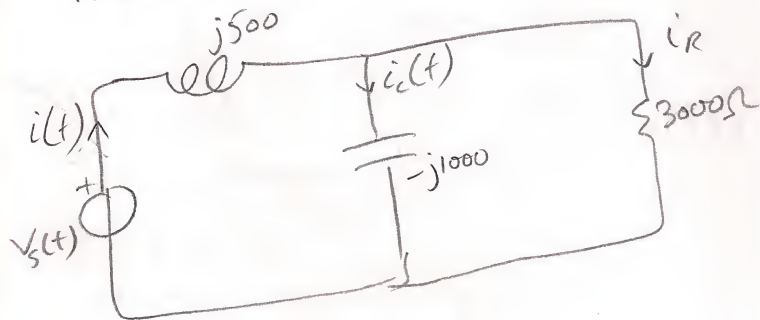
$$i_c(t) = 0.055 \cos(2000t - 71.56) \text{ A}$$

12.29 $i(t), i_c(t), i_R(t)$

$$v_s(t) = 100 \cos(2000t) \text{ V}$$

$$L = 250 \text{ mH} \quad C = 0.5 \mu\text{F}$$

$$R = 3 \text{ k}\Omega$$



$$R_{11C} = \frac{3000(-j1000)}{3000 - j1000} = \frac{-j3000}{3-j} \times \frac{3+j}{3+j}$$

$$R_{11C} = \frac{-9000j + 3000}{9+1} = \frac{-9000j + 3000}{10}$$

$$R_{11C} = 300 - 900j$$

$$v(R_{11C})(t) = \frac{300 - 900j}{300 - 900j + j500} \times 100$$

$$= \frac{300 - 900j}{300 - 400j} \times 100$$

$$= \frac{300 - 900j}{3 - 4j} \times \frac{3 + 4j}{3 + 4j}$$

$$v(t) = \frac{900 + 3600 + 1200j - 2700j}{9 + 16}$$

$$= \frac{4500 - 1500j}{25}$$

$$v(t) = 180 - 60j \leftarrow (R_{11C})$$

$$i_c(t) = \frac{180 - 60j}{-j1000} \times \frac{j1000}{j1000}$$

$$= \frac{180000j + 60000}{(1000)(1000)}$$

$$i_c(t) = 0.06 + 0.18j$$

$$= \cancel{0.24}$$

$$= 0.189 \angle 71.57^\circ$$

$$i_c(t) = 0.189 \cos(2000t + 71.57)$$

$$i_R(t) = \frac{180 - 60j}{3000}$$

$$= 0.06 - 0.02j$$

$$= 0.0632 \angle -18.43$$

$$i_R(t) = 0.0632 \cos(2000t - 18.43)$$

$$i_c(t) = \frac{(300 - 900j)(\angle \times V(t))}{(300 - 900j) + j500}$$

$$= \frac{(300 - 900j)}{300 - 400j}$$

$$= \frac{(300 - 900j)(3 + 4j)}{9 + 16}$$

$$= \frac{900 + 1200j - 2700j + 3600}{25}$$

$$= \frac{4500 - 1500j}{25}$$

$$V_{LR}(t) = 180 - j60$$

$$i(t) = \frac{180 - j60}{-j1000}$$

$$= \frac{60 + 18j}{1000}$$

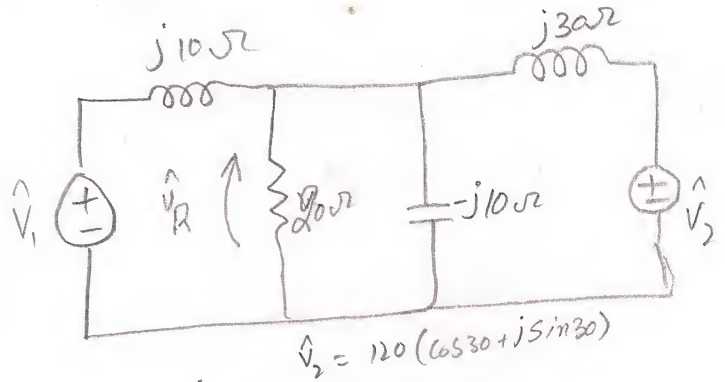
$$= \frac{6}{1000} + \frac{18}{1000}j = 0.06 + 0.018j$$

$$i_R(t) = \frac{180 - j60}{3000}$$

$$= 0.06 - j0.02$$

$$i_t = 0.06 + 0.18j + 0.06 - j0.02$$

$$= 0.12 + 0.16j$$



$$Z_L = j10\Omega, \quad = 120$$

$$\hat{V}_R = \frac{-j100}{j10} + \frac{(103.92 + j60)j}{j30}$$

$$\frac{-j}{10} + \frac{1}{20} + \frac{+j}{10} + \frac{-j}{30}$$

$$\frac{1}{j10} = -j10 + \frac{103.92 + j60}{j30}$$

$$\frac{-6j + 30 + 6j - 2j}{60}$$

$$= \frac{-j300 - j103.92 + 160}{3 - 2j}$$

$$= -2j(-j300 - j103.92 + 160)$$

$$= -600 - 207.84 - 120j$$

$$= -807.84 - 120j$$

$$V(t) = \frac{50}{-\frac{1}{j500} + \frac{1}{5+j100}} \text{ mV}$$

$$= \frac{50(j500)(5+j100)}{-(5+j100)+j500} \text{ mV}$$

$$= \frac{2500j(5+j100)}{-5-j100+j500} \text{ mV}$$

$$= \frac{125j - 250000}{j400 - 5} \times \frac{j400+5}{j400+5}$$

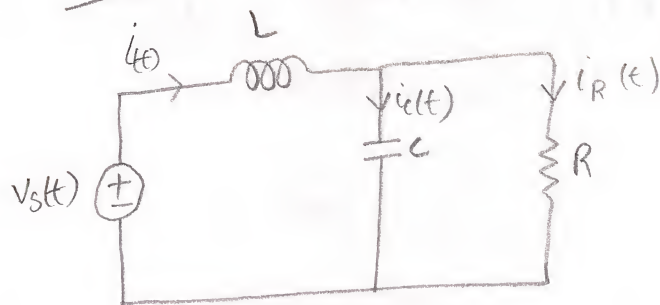
$$= \frac{-50000 + 625j - 100000000j - 1250000}{160000 + 25}$$

$$= \frac{-1300000 - 9999375j}{160025}$$

$$= -8.12 - j62.48$$

=

12.29



$$V_s(t) = 100 \cos(2000t) \text{ V}$$

$$V_s(t) = 100 e^{j\omega t} = 100 \angle 0^\circ$$

$$\omega = 2000$$

$$Z_L = j2000 \times 250 \times 10^{-3}$$

$$= j500000 \Omega$$

$$Z_C = \frac{-j \times 10^6}{200 \times 0.5} = -j10000$$

$$R = 3000 \Omega$$

$$Z_{eq} = Z_L + \frac{R Z_C}{R + Z_C}$$

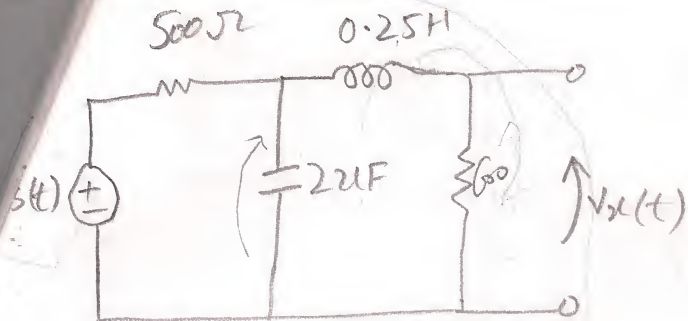
$$= R_L + \frac{3000 \times -j10000}{3000 + (-10000j)}$$

$$= Z_L + \frac{-3000000j}{3000 - 10000j} \times \frac{3000 + 10000j}{3000 + 10000j}$$

$$= Z_L + \frac{-9000000000j + 3000000000}{3000000 + 100000000}$$

$$= j500 + 300 - 900j \rightarrow \underline{\underline{R_{ILC}}}$$

$$= 300 - 400j$$



$$Z_C = \frac{-j \times 10^6}{1000 \times 2} = -j500 \Omega$$

$$Z_L = j1000 \times \frac{1}{4} = j250 \Omega$$

$$V_x = \frac{5}{500} \quad \text{(with a circled } \frac{600}{12} \text{ next to it)}$$

$$\frac{1}{500} + \frac{1}{-j500} + \frac{1}{600 + j250}$$

$$= \frac{5}{-j500 + 500 + \frac{600 - j250}{600^2 + 1250}} \text{ mV}$$

$$= \frac{+500(j-1)}{+500j500} + \frac{600 - j250}{36000 + 62500}$$

$$= \frac{j-1}{j500} + \frac{600 - j250}{42250} \text{ mV}$$

$$= \frac{j-1}{j500} + \frac{0.142 - j.00591}{5(j500)}$$

$$= \frac{j-1 + j71 + j2.955}{5} \text{ mV}$$

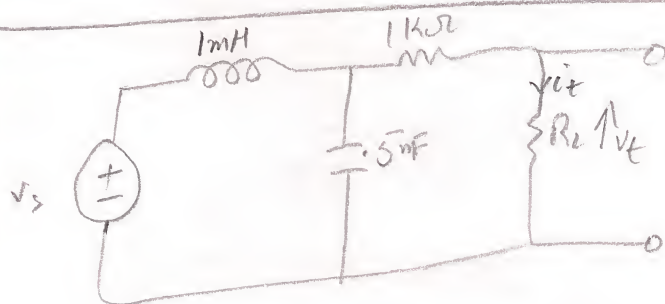
$$= \frac{2500j}{-1 + j74.955} \text{ mV}$$

$$= \frac{(2500j)(-1 - j74.955)}{1 + 74.955^2}$$

$$= \frac{-2500j + 187387.5}{5619.25}$$

$$= \frac{-2500j + 33.35}{5619.25}$$

$$= -0.44 + j33.35$$



$$V_s = 30 \angle 0^\circ$$

$$X_L = j10^{-3} \times 10^6 = j1000 \Omega$$

$$X_C = \frac{-j}{\frac{6-9}{10 \times 10^6 \times 5}} = -j \frac{10^3 \times 10}{5} = -j2000 \Omega$$

$$Z_{eq} = 1000 + \frac{j1000 \times -j2000}{j1000 - j2000}$$

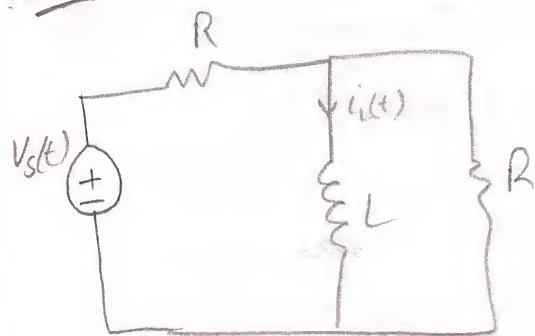
$$= 1000 + \frac{j1000 \times -j2000}{j1000(1 - 2)}$$

$$= 1000 + \frac{+j2000}{+1}$$

$$= 1000 + j2000$$

$$X_L = j10^3$$

12.31



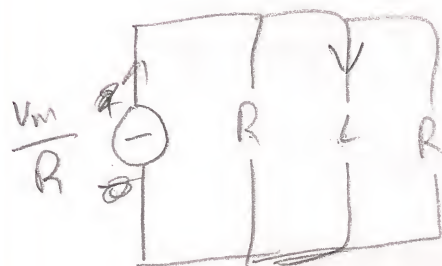
$$V_s(t) =$$

$$i_L(t) = \frac{2R}{2R+L} \times \frac{V_s(t)}{R}$$

$$= \frac{2V_s(t)}{2R+L}$$

$$= \frac{2V_m}{2R+j\omega L} \times \frac{2R-j\omega L}{2R-j\omega L}$$

$$= \frac{4RV_m - j2V_m\omega L}{4R^2 + \omega^2 L^2}$$



$$\frac{\frac{R}{2}}{\frac{R}{2} + L} \times \frac{V_m}{R}$$

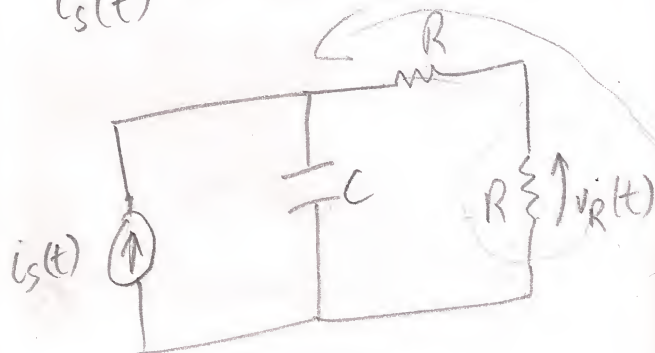
$$= \frac{V_m}{R+2L}$$

$$i_t = \frac{V_m}{R+j2\omega L}$$

12.32

$$i_s(t) = i_m \angle -90^\circ$$

$$i_s(t) = i_m(-j) = -ji_m$$



$$V_R = \frac{1}{2} \left(\frac{-ji_m}{\frac{1}{j\omega C} + \frac{1}{2R}} \right)$$

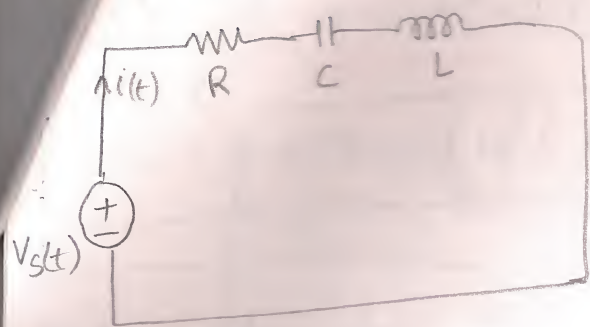
$$= \frac{1}{2} \frac{-ji_m}{\frac{2R+j\omega L}{2Rj\omega C}}$$

$$= \frac{\frac{1}{2} \times 2Rj\omega C \times (-ji_m)}{2R+j\omega L}$$

$$= \frac{-jR(j\omega C)i_m}{2R+j\omega L}$$

$$= \frac{-R \frac{1}{j\omega C} i_m}{\frac{j2R\omega C + 1}{j\omega C}} = \frac{-jRi_m}{1+j2R\omega L}$$

$$= \frac{-jRi_m}{1+j2R\omega L} \angle -\tan^{-1} 2R\omega L$$



$$V_s = 35 \cos(1000t)$$

$$= 35 e^{j0}$$

$$= 35 \text{ V}$$

$$Z_L = j 1000 \times 25 \times 10^{-3} = j 25 \Omega$$

$$Z_C = \frac{-j}{1000 \times 10 \times 10^{-6}} = -j 10^2$$

$$= -j 100 \Omega$$

$$R = 50 \Omega$$

$$i(t) = \frac{35}{50 - j 100 + j 25} \text{ A}$$

$$= \frac{35}{50 - j 75} \times \frac{50 + j 75}{50 + j 75}$$

$$= \frac{1750 + j 2625}{2500 + 5625}$$

$$= 0.215 + j 0.323$$

$$|i| = 0.388$$

$$\angle i = \tan^{-1} \frac{0.323}{0.215}$$

$$= 56.35$$

$$\hat{i} = \frac{0.388 \angle 56.35}{\sqrt{2}}$$

$$= 0.27 \angle 56.35$$

$$\hat{V}(R) = 0.27 \angle 56.31^\circ \times 50$$

$$= 13.728 \angle 56.31$$

$$\hat{V}(C) = 0.27 \angle 56.31^\circ \times (-j 100)$$

$$= 0.27 (\cos 56.31 + j \sin 56.31) (-j 100)$$

$$= 0.27 (0.55 + j 0.83) (-j 100)$$

$$= 0.27 (-j 55 + 83)$$

$$= 22.41 - j 14.85$$

$$\hat{V}(C) = 26.88 \angle -33.53$$

$$\hat{V}(L) = 0.27 \angle 56.31^\circ \times j 25$$

$$= 0.27 (0.55 + j 0.83) j 25$$

$$= 0.27 (j 13.75 - 20.75)$$

$$= -5.625 + j 3.7125$$

$$= 6.718 \angle 146.469^\circ$$

$$V(t) = \frac{300j - 125}{85j - 85} \times \frac{-85 - 85j}{-85 - 85j}$$

$$= \frac{-25500j + 10625j + 10625 + 25500}{14450}$$

$$= \frac{36125 - 14875j}{14450}$$

$$V(t) = 2.5 - 1.029j$$

$$V_x(t) = \frac{600}{600 + j250} \times (2.5 - 1.029j)$$

$$= \frac{1500 - 617.4j}{600 + j250} \times \frac{600 - j250}{600 - j250}$$

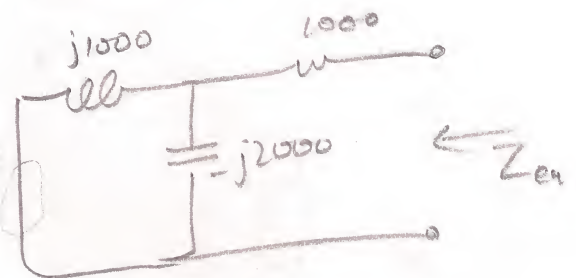
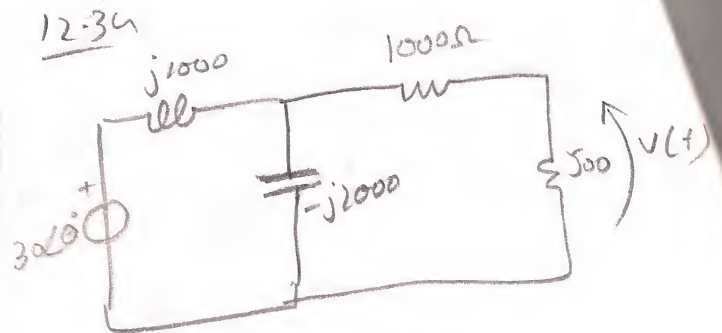
$$= \frac{-370440j - 375000j + 900000 - 154375}{422500}$$

$$= \frac{745625 - 745440j}{422500}$$

$$V_n(t) = 1.7647 - 1.764j$$

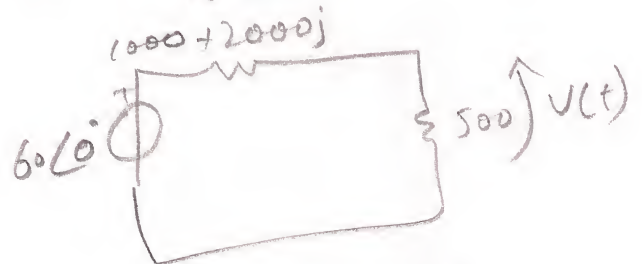
$$= 2.49 \angle -45^\circ$$

$$V_n(t) = 2.49 \cos(1000t - 45^\circ) V$$



$$Z_{th} = \frac{2 \times 10^3}{-j1000} = 2000j + 1000$$

$$V_{oc} = \frac{-j2000}{-j1000} \cdot 30 = 60 \angle 0^\circ$$



$$V(t) = \frac{500}{1500 + 2000j} \cdot 60$$

$$= \frac{60}{3 + 4j} \times \frac{3 - 4j}{3 - 4j}$$

$$= \frac{180 - 240j}{9 + 16}$$

$$V(t) = 3.529 - 14.1176j$$

$$V(t) = 14.55$$

$$= 7.2 - 4.6j$$

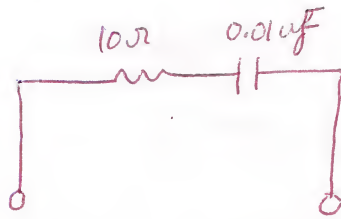
$$= 12 \angle -53.13^\circ$$

$$V(t) = 12 \cos(10^6 t - 53.13^\circ)$$

Solving STARS

12.10

$\phi = ?$



$$\phi = 12.5^\circ = \angle V_R + \angle i \quad \left\{ \begin{array}{l} \text{Current will} \\ \text{lead.} \end{array} \right.$$

Solution

$$Z = 10 + \frac{-j \times 10^6}{2\pi f \times 0.01}$$

$$= \left(10 - j \frac{15915494.31}{f} \right) \Omega$$

$$\angle Z = \angle V - \angle i = -12.5^\circ$$

So

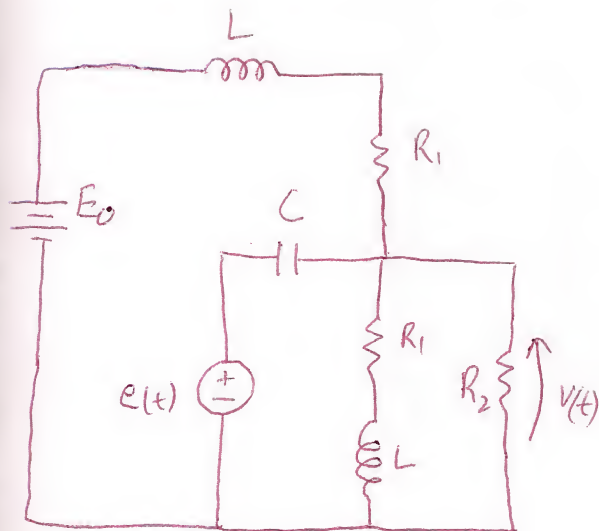
$$-12.5^\circ = \tan^{-1} \left(\frac{-\frac{15915494.31}{f}}{10} \right)$$

$$+0.22 = + \frac{15915494.31}{10f}$$

$$f = \frac{15915494.31}{2.2} = \boxed{7.23 \text{ MHz}}$$

12.24

Find $V(t)$,



$$E_0 = 10V$$

$$f = 1000 \text{ Hz}$$

$$e(t) = 2 \sin(\omega t + 30^\circ)$$

$$L = 1 \text{ mH}$$

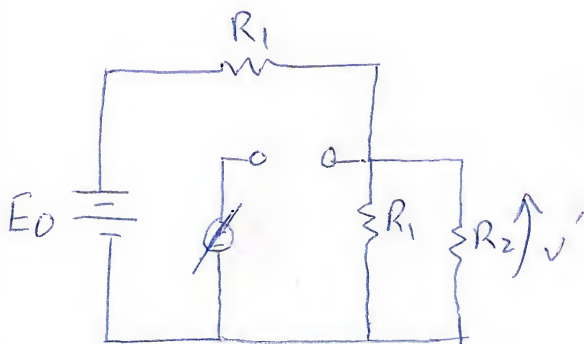
$$C = 0.5 \mu\text{F}$$

$$R_1 = 100 \Omega, R_2 = 2200 \Omega$$

Solution

Super-Position

i) $V(t)$ According to $E_0 = V'$



MILLMANN

$$V' = \frac{10/100}{\frac{1}{100} + \frac{1}{100} + \frac{1}{2200}}$$

$$V' = \frac{10}{2 + \frac{1}{2.2}} = \frac{22}{5.45} = 4.07V$$

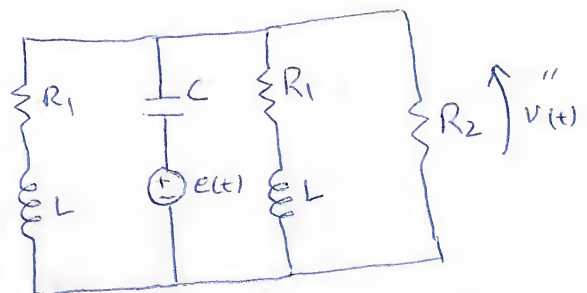
ii) V'' (According to $e(t)$)

$$e(t) = 2 \angle -60^\circ = (1 - \sqrt{3}j) V$$

$$Z_L = j62.83 \Omega$$

$$Z_C = -j318 \Omega$$

This is the mistake.
which changes the whole Result. Method is Right.



$$V'' = \frac{1 - \sqrt{3}j}{-j318} \cdot \frac{2}{\frac{1}{100 + j62.83} + \frac{1}{j318} + \frac{1}{2200}}$$

$$= \frac{\frac{1 - \sqrt{3}j}{-j318}}{\frac{1}{2200} + \frac{1}{(100 + j62.83)(-j318)}}$$

$$= \frac{(1 - \sqrt{3}j)(2200)(100 + 62.83j)}{-j1399200 - 220000 - j138226 - j31800 + 19980}$$

$$V''(t) = \frac{220000 + j138226 - j381051 + 234414.455}{-200020 - j1569226}$$

$$= \frac{459414.545 - j242825}{-200020 - j1569226} \Rightarrow \frac{519639.98 \angle -27.85^\circ}{1569353.7 \angle -90.73^\circ}$$

$\theta - 180 \Rightarrow -90.73^\circ$

$$V''(t) = (0.33 \angle +62.88^\circ) V$$

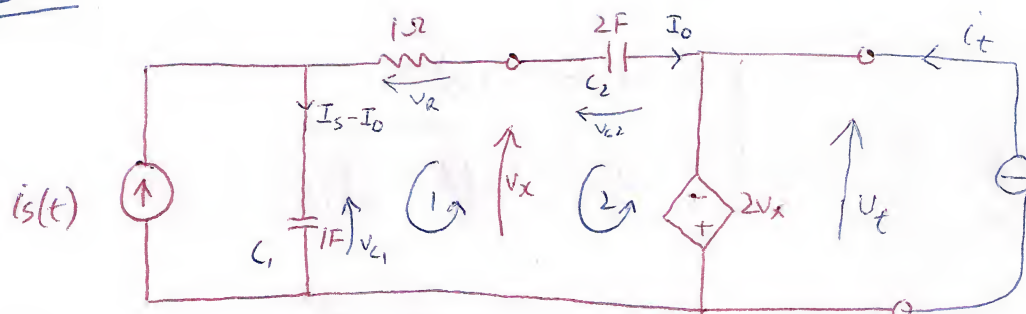
So FINALLY

#ERROR
ANSWER NOT
VERIFIED

$$V(t) = V' + V''(t)$$

$$= \left[4.07 + 0.33 \cos(\omega t + 62.88^\circ) \right] V$$

12.26



$$i_s(t) = 10 \sin(\omega t + 30^\circ) A$$

$$\omega = 314 \text{ rad/s}$$

Solution

$$i_s(t) = (5 - j8.66) A$$

$$Z_{C1} = -j/314 \Omega$$

$$Z_{C2} = -j/628 \Omega$$

$$V_t + 2V_x = 0$$

$$V_t = -2V_x \Rightarrow V_x = -\frac{V_t}{2} \quad \text{--- (i)}$$

KVL1

$$V_x + I_0 \left(-\frac{j}{314} \right) (I_s - I_0) = 0$$

$$V_x + I_0 \left[\frac{j}{314} \times (5 - j8.66 - I_0) \right] = 0$$

$$I_0 \left(1 - \frac{j}{314} \right) = -V_x - \left(\frac{8.66}{314} + \frac{5}{314} j \right)$$

$$I_0 = \frac{-314V_x - 8.66 - 5j}{314 - j} \quad \text{--- (ii)}$$

$$\text{KVL}^2 \quad I_0 \left(\frac{-j}{628} \right) - V_x - 2V_x = 0$$

Putting value from i, ii \Rightarrow

$$3 \left(-\frac{V_t}{2} \right) = \frac{-j}{628} \left(\frac{-314 \left(-\frac{V_t}{2} \right) - 8.66 - 5j}{314 - j} \right)$$

$$-\frac{3}{2}V_t = \frac{-157jV_t}{2 \times 314(628 - 2j)} + \frac{8.66j}{314(628 - 2j)} - \frac{5}{314(628 - 2j)}$$

$$\frac{5}{314(628 - 2j)} - \frac{8.66j}{314(628 - 2j)} = V_t \left(\frac{3}{2} - \frac{157j}{314(628 - 2j)} \right)$$

$$\frac{5 - 8.66j}{314(628 - 2j)} = V_t \left(\frac{3(314(628 - 2j)) - 314j}{2 \times 314(628 - 2j)} \right)$$

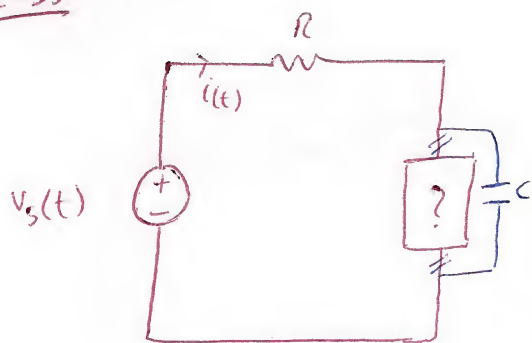
$$V_t = \frac{10 - 17.32j}{591576 - j2198} = \frac{20 \angle -60}{591580 \angle -0.21}$$

$$= 3.38 \times 10^{-5} \angle -59.8^\circ$$

$$V(t) = V_{\text{eff}} = \left[3.38 \times 10^{-5} \cos(\omega t - 59.8^\circ) \right] V$$

$$Z_{\text{eq}} = 0 \Omega$$

12-35



$$\angle i - \angle V = 55^\circ \Rightarrow$$

$$V_s(t) = 120\sqrt{2} \cos(120\pi t) \text{ V}$$

$$R = 10 \Omega$$

(My Answer was Right)

Solution

i) As current is leading the phase of voltage so the unknown ELEMENT is a capacitor.

ii) $C = ?$

$$Z_{eq} = 10 + \frac{1}{j\omega C}$$

$$\angle Z_{eq} = -55^\circ = \tan^{-1}\left(\frac{-\frac{1}{\omega C}}{10}\right)$$

$$+1.2428 = +\frac{1}{10\omega C} \quad \left\{ \omega = 120\pi \text{ rad/s} \right\}$$

$$C = \frac{1}{10 \times 120\pi \times 1.2428} = \boxed{186 \mu\text{F}} \quad \checkmark \Rightarrow Z_C = \frac{-j}{\omega C} = \frac{-j10}{186 \times 120\pi} = -j14.26 \Omega$$

ii) $i(t)_p = ?$ — Peak value is $\hat{V}(t) = \frac{V_p}{\sqrt{2}} e^{j\theta}$
 \Downarrow
 The magnitude of the phasor.

$$\text{So } i(t) = \frac{120\sqrt{2} \angle 0^\circ}{R + Z_C} = \frac{120\sqrt{2} \angle 0^\circ}{10 - j14.26}$$

$$i(t) = \left(9.74 \angle 54.96^\circ \right) \text{ A}$$

$$= 9.74 e^{j54.96^\circ} \text{ A} \Rightarrow$$

$$\text{So } I_p = 9.74 \text{ A}$$

But Given Answer is: $\frac{9.74}{\sqrt{2}} = 6.88 \text{ A}$
 ?

if we Assume
 $V_s(t) = 120 \cos(120\pi t) \text{ V}$
 AS Result
 $I_p = 6.88 \text{ A}$
 VERIFIED

CH # 12

Rectangular Form: $\hat{x} = a + jb$

$$\text{Magnitude} = \sqrt{a^2 + b^2} = |\hat{x}|$$

$$\text{Phase Angle} = \angle \hat{x} = \tan^{-1} \frac{b}{a}$$

$$\text{if } \hat{x} = a + jb, \quad \tan^{-1} \frac{b}{a} = \theta^\circ$$

$$\hat{x} = a - jb, \quad \tan^{-1} \frac{-b}{a} = \theta^\circ$$

$$\hat{x} = -a + jb, \quad \left(\tan^{-1} \frac{b}{-a}\right) + 180^\circ = \theta^\circ$$

$$\hat{x} = -a - jb, \quad \left(\tan^{-1} \frac{b}{-a}\right) - 180^\circ = \theta^\circ$$

Phasor Form $\hat{x} = |\hat{x}| \angle \theta^\circ$

$$\hat{x} = |\hat{x}| e^{j\theta} \quad (\text{Polar Form})$$

$$\text{Sinusoidal Form} = |\hat{x}| \cos(\omega t + \theta^\circ) = \hat{x} \quad (i)$$

$$\omega = \text{Angular Frequency} = 2\pi f$$

$$\angle = \theta^\circ$$

$$\hat{x} = |\hat{x}| \sin(\omega t + \theta^\circ) \quad (ii)$$

$$\angle = \theta^\circ - 90^\circ$$

CH # 13

If we use R.M.S values OR Phasor values

$$\text{Active Power } P = |\hat{V}| |\hat{I}| \cos \theta \quad W$$

$$\text{Reactive Power } Q = |\hat{V}| |\hat{I}| \sin \theta \quad \text{KVAR}$$

$$\text{APPARENT Power } A = |\hat{V}| |\hat{I}| \quad \text{VA}$$

$$\text{Complex Power } S = \hat{V} \hat{I}^*$$

$$= P + jQ \quad \left. \begin{array}{l} \\ \\ \end{array} \right\} \text{VA}$$

$$= A e^{j\theta}$$

$$\angle S = \angle Z = \angle V - \angle i$$

$$P = \frac{1}{2} |\hat{V}| |\hat{I}| \cos \theta \quad W$$

$$Q = \frac{1}{2} |\hat{V}| |\hat{I}| \sin \theta \quad \text{VAR}$$

$$A = \frac{1}{2} |\hat{V}| |\hat{I}| \quad \text{VA}$$

$$S = \frac{1}{2} \hat{V} \hat{I}^* \quad \text{VA}$$

$$= A e^{j\theta}$$

$$= P + jQ$$

$$\angle S = \angle Z = \angle \hat{V} - \angle \hat{i}$$

$$X_{rms} = \frac{\hat{x}}{\sqrt{2}}$$

$$\theta = \theta_v - \theta_i$$

$$v(t) = 168 \cos(377t + 45^\circ) \text{ V}$$

$$i(t) = 0.88 \cos(377t) \text{ A}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\phi_v - \phi_i)$$

$$= \frac{168}{\sqrt{2}} \cdot \frac{0.88}{\sqrt{2}} \cos(45^\circ - 0^\circ) \checkmark$$

$$P = 73.92 (0.707)$$

$$P = 52.269 \text{ W}$$

Since it is +ve
so element absorbs
energy.

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\phi_v - \phi_i) \checkmark$$

$$= \frac{168 \times 0.88}{2} \sin(45^\circ - 0^\circ)$$

$$= 73.92 (0.707)$$

$$Q = 52.27 \text{ VAR}$$

$$b) v(t) = 285 \cos(2500t - 68^\circ) \text{ V}$$

$$i(t) = 0.66 \cos(2500t) \text{ A}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(-68^\circ - 0^\circ)$$

$$P = \frac{285 \cdot 0.66}{2} \cos(-68^\circ)$$

$$= 94.08 (0.374)$$

$$P = 35.24 \text{ W}$$

Absorbs energy

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(-68^\circ - 0^\circ)$$

$$= 94.08 (-0.927)$$

$$Q = -87.229 \text{ VAR}$$

$$c) v(t) = 168 \cos(377t + 45^\circ) \text{ V} \quad (1)$$

$$i(t) = 0.88 \cos(377t - 60^\circ) \text{ A}$$

$$P = \frac{168}{\sqrt{2}} \cdot \frac{0.88}{\sqrt{2}} \cos(45^\circ + 60^\circ)$$

$$= 73.92 \cos(105^\circ)$$

$$P = -19.13 \text{ W}$$

-ve \Rightarrow supplies energy

$$Q = \frac{168}{\sqrt{2}} \cdot \frac{0.88}{\sqrt{2}} \sin(105^\circ)$$

$$= 73.92 (0.966)$$

$$Q = 71.401 \text{ VAR}$$

$$d) v(t) = 285 \cos(2500t - 68^\circ) \text{ V}$$

$$i(t) = 0.8 \sin(2500t) \text{ A}$$

$$P = \frac{285}{\sqrt{2}} \cdot \frac{0.8}{\sqrt{2}} \cos(-68^\circ + 90^\circ)$$

$$= 125.4 (0.3746)$$

$$P = 46.70 \text{ W}$$

$$P = 116.268 \text{ W}$$

absorbs energy

$$Q = \frac{285}{\sqrt{2}} \cdot \frac{0.8}{\sqrt{2}} \sin(-68^\circ + 90^\circ)$$

$$= 125.4 (0.374)$$

$$Q = 46.97 \text{ VAR}$$

(REMEMBER These Questions MAY vary, But
METHOD is SAME)

13.2

$$a) \hat{V} = 120 \angle 30^\circ \text{ V} \quad \hat{I} = 20 \angle 75^\circ \text{ A}$$

$$Z = \frac{120 \angle 30^\circ}{20 \angle 75^\circ} = 6 \angle -45^\circ \Omega$$

$$Z = 6 e^{-j45^\circ} \Omega$$

$$b) A = 3.3 \text{ kVA} \quad Q = -1.8 \text{ kVAR}$$

$$|\hat{I}| = 7.5 \text{ A} \Rightarrow I_{\text{rms}} = 7.5 \text{ A}$$

$$Q = V_{\text{rms}} \cdot I_{\text{rms}} \sin(\phi_v - \phi_i)$$

$$\frac{-1.8 \times 10^3}{7.5} = V_{\text{rms}} \sin(\phi_v - \phi_i)$$

$$V_{\text{rms}} \sin(\phi_v - \phi_i) = -240$$

$$A = V_{\text{rms}} I_{\text{rms}}$$

$$\frac{3.3 \times 10^3}{7.5} = V_{\text{rms}} \Rightarrow V_{\text{rms}} = 440 \text{ V}$$

$$\sin(\phi_v - \phi_i) = \frac{-240}{440}$$

$$(\phi_v - \phi_i) = \sin^{-1}(-0.545)$$

$$\phi_v - \phi_i = -33.055^\circ$$

$$Z = \frac{440}{7.5} e^{-j33.055^\circ}$$

$$Z = 58.66 e^{-j33.055^\circ} \Omega$$

$$c) P = 3 \text{ kW} \quad Q = 4 \text{ kVAR} \quad \hat{V}$$

$$V_{\text{rms}} = 80 \text{ V}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\phi_v - \phi_i)$$

$$\frac{3000}{80} = I_{\text{rms}} \cos(\phi_v - \phi_i)$$

$$I_{\text{rms}} \cos(\phi_v - \phi_i) = 3.409 \quad \text{--- } \odot$$

$$Q = V_{\text{rms}} I_{\text{rms}} \sin(\phi_v - \phi_i)$$

$$\frac{4000}{80} = I_{\text{rms}} \sin(\phi_v - \phi_i)$$

$$I_{\text{rms}} \sin(\phi_v - \phi_i) = 4.545$$

$$\tan(\phi_v - \phi_i) = 1.33$$

$$\phi_v - \phi_i = 53.13^\circ \quad \text{pol. in } \odot$$

$$I_{\text{rms}} = \frac{3.409}{0.599} \Rightarrow I_{\text{rms}} = 5.68 \text{ A}$$

$$Z = \frac{80}{5.68} e^{j53.13^\circ}$$

$$Z = 154.88 e^{j53.13^\circ} \Omega$$

$$d) |\hat{V}| = 208 \text{ V} \quad |\hat{I}| = 17.8 \text{ A}$$

$$V_{\text{rms}} = 208 \text{ V}$$

$$I_{\text{rms}} = 17.8 \text{ A}$$

$$P = V_{\text{rms}} I_{\text{rms}} \cos(\phi_v - \phi_i)$$

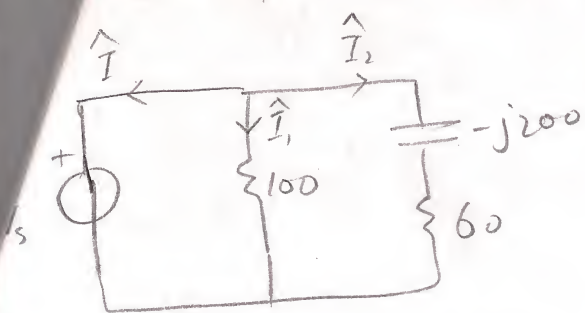
$$\cos(\phi_v - \phi_i) = \frac{3000}{208 \cdot 17.8} = 0.81028$$

$$\phi_v - \phi_i = \pm 35.87^\circ$$

$$Z = 11.68 e^{\pm j35.87^\circ}$$

$$\hat{V}_s = 15 \angle 0^\circ, R_1 = 100 \Omega$$

$$R_2 = 60 \Omega, Z_C = -j200 \Omega$$



$$\hat{I}_2 = \frac{15 \angle 0^\circ}{60 - j200} = \frac{15 \angle 0^\circ}{200.80 \angle -73.30^\circ}$$

$$\hat{I}_2 = 0.0718 \angle 73.30^\circ \text{ A} \checkmark$$

$$\hat{I}_1 = \frac{15 \angle 0^\circ}{100} = 0.15 \angle 0^\circ \text{ A} \checkmark$$

$$S_2 = \hat{V}_s \hat{I}_2^* = 15 e^{j0} \cdot 0.0718 e^{-j73.30}$$

$$S_2 = 1.077 e^{-j73.30} \text{ VA}$$

$$S_1 = \hat{V}_s \hat{I}_1^* = 15 e^{j0} \cdot 0.15 \angle 0^\circ$$

$$S_1 = 2.25 \text{ VA}$$

$$b) \hat{I} = -(\hat{I}_1 + \hat{I}_2)$$

$$= -(0.15 + 0.02063 + j0.06877)$$

$$\hat{I} = -0.17063 - j0.06877$$

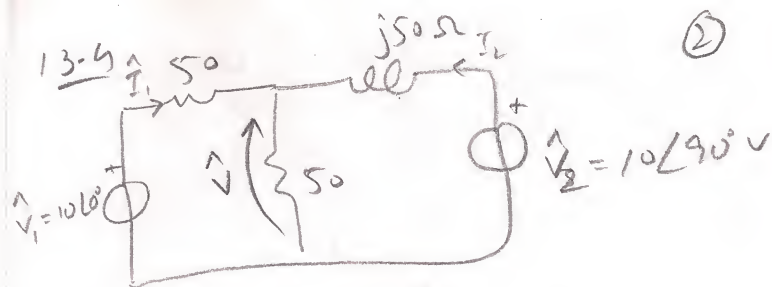
$$S = \hat{V}_s \hat{I}^* = 15 \angle 0^\circ \cdot 0.1839 \angle +58.04^\circ$$

$$S = 2.7585 e^{j58.04} \text{ VA}$$

$$PF = \cos(\phi)$$

$$PF = 0.93$$

$$\cos(\phi_1 - \phi_2) = \cos(0 - 73.3) = 0.93$$



$$\hat{V} = \frac{\frac{j10}{50} + \frac{10}{50}}{\frac{1}{50} + \frac{1}{50} + \frac{1}{j50}}$$

$$\hat{V} = \frac{\frac{2}{5}}{\frac{2j+1}{j50}} = \frac{20j}{2j+1} \times \frac{1-2j}{1-2j}$$

$$\hat{V} = \frac{20j+40}{5} = 8+4j \text{ V}$$

$$\hat{I}_1 = \frac{10 - (8+4j)}{50}$$

$$\hat{I}_1 = \frac{2-4j}{50} = \frac{1-2j}{25} \text{ A}$$

$$S_1 = 10 \hat{I}_1 = 0.04 - 0.08j \text{ A}$$

$$= 0.089 \angle -63.43^\circ$$

$$S_1 = 10 \angle 0^\circ \cdot 0.089 \angle +63.43^\circ \checkmark$$

$$S_1 = 0.89 e^{j63.43}$$

$$S_1 = (0.39 + j0.796) \text{ VA}$$

$$S_2 = \hat{V}_2 \cdot \hat{I}_2 = \hat{V}_2 + \frac{\hat{V}_2^*}{j50}$$

$$S_2 = 10 \angle 90^\circ + \frac{10 \angle -90^\circ}{j50}$$

$$= \frac{100}{j50} + \frac{-j100}{j50}$$

$$= -5000j + 2000$$

$$\hat{I}_L = \frac{j10 - (8+4j)}{j50}$$

$$\hat{I}_L = \frac{-8+6j}{j50} \times \frac{-j50}{-j50}$$

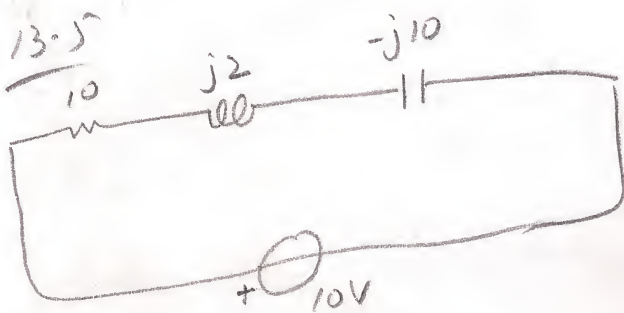
$$\hat{I}_L = \frac{400j + 300}{2500} = \frac{3+4j}{25}$$

$$\hat{I}_L = 0.12 + 0.16j$$

$$S_L = \hat{V}_L \cdot \hat{I}_L^*$$

$$= j10 \cdot (0.12 - 0.16j)$$

$$S_L = 1.6j + 1.2 \text{ VA}$$



$$I_{rms} = \frac{10}{10+j2-j10} = \frac{10}{10-j8} = \frac{5}{5-j4} \text{ A}$$

$$I_{rms} = \frac{5(5+j4)}{25+16} = \frac{25+20j}{41}$$

$$I_{rms} = 0.609 + 0.487j$$

$$= 0.7802 \angle 38.64^\circ$$

$$P = V_{rms} \cdot I_{rms} \cos(0 - 38.64^\circ)$$

$$= 7.802(0.7810)$$

$$P = 6.094 \text{ W}$$

$$13.6 \quad \hat{V}_L = 80 \text{ V}$$

$$\hat{I} = 12 \text{ A}$$

$$S_A = V_{rms} I_{rms} = 960 \text{ VA}$$

$$b) \text{ PF} = \cos(\phi_v - \phi_i) = \cos(25.5^\circ)$$

$$\text{PF} = 0.9025$$

$$c) P = V_{rms} I_{rms} \cos(\phi_v - \phi_i)$$

$$P = 866.48 \text{ W}$$

$$d) Q = V_{rms} I_{rms} \sin(\phi_v - \phi_i)$$

$$= 460(0.4305)$$

$$Q = 413.29 \text{ VAR}$$

$$e) Z_{im} = \frac{\hat{V}}{\hat{I}} \angle 25.5^\circ$$

$$= 6.67 e^{j25.5^\circ}$$

$$= 6.67 (\cos(25.5^\circ) + j \sin(25.5^\circ))$$

$$Z_{im} = 6.019 + 2.87j \Omega$$

$$13.7 a) S = (1000 + j750) \text{ VA}$$

$$S = 1250 \angle 36.869^\circ$$

$$\text{PF} = \cos(36.869^\circ)$$

$$\text{PF} = 0.8$$

since $\phi = +ve$
so it is inductive

$$b) |\hat{V}| = 440 \text{ V}, |Z_L| = 30 \Omega, P = 3 \text{ kW}$$

$$P = V_{rms} I_{rms} \cos(\phi_v - \phi_i)$$

$$I_{rms} = \frac{V_{rms}}{Z_L} \Rightarrow I_{rms} = 14.67 \text{ A}$$

$$3000 = 440 \cdot 14.67 \cos(\phi_v - \phi_i)$$

$$(\phi_v - \phi_i) = 0.4647$$

$$(\phi_v - \phi_i) = \arccos(0.4647)$$

$$(\phi_v - \phi_i) = \pm 62.30$$

Not possible to say about load

$$\boxed{PF = 0.46}$$

$$c) A = 10 \text{ kVA}, Q = -8 \text{ kVA} \quad P > 0$$

$$A = V_{rms} \cdot I_{rms} \quad Q = V_{rms} \cdot I_{rms} \sin(\phi_v - \phi_i)$$

$$10000 = V_{rms} \cdot I_{rms} \quad \frac{-8000}{10000} = \sin(\phi_v - \phi_i)$$

$$\phi_v - \phi_i = \arcsin(-0.8)$$

$$\boxed{\phi_v - \phi_i = -53.13^\circ}$$

capacitive
due to -ve angle

$$\boxed{PF = 0.6}$$

$$13.8 \quad V_{rms} = 440 \text{ V}, \quad A = 3 \text{ kVA}$$

$$PF = 0.9 \quad (\text{inductive})$$

$$I_{rms} = ? \quad P, Q = ?$$

$$PF = 0.9$$

$$\phi_v - \phi_i = \arccos(0.9)$$

$$\boxed{\phi_v - \phi_i = +25.84^\circ}$$

$$A = V_{rms} \cdot I_{rms} \Rightarrow I_{rms} = \frac{3000}{440}$$

$$\boxed{I_{rms} = 6.81 \text{ A}}$$

$$P = 3000 \times 0.9 \Rightarrow \boxed{P = 2700.02 \text{ W}}$$

$$Q = 3000 (+0.4358)$$

③

$$\boxed{Q = +1307.62 \text{ VAR}}$$

13.9



$$P(Z) = 8 \text{ W}$$

$$P(V_{in}) = 8 \text{ W}$$

$$V_{in} \cdot I_{rms} \cos(\phi_v - \phi_i) = 8$$

$$\frac{V_{in}(t)}{12} \cos(\phi_v - \phi_i) = 8$$

$$V_{in}(t) \cos(\phi_v - \phi_i) = 96$$

13.10

$$V_{rms} = 2400 \text{ V}$$

$$A = 10 \text{ kVA}$$

$$PF = 0.8 \quad (\text{inductive})$$

$$\boxed{\phi_v - \phi_i = 36.86^\circ}$$

$$\frac{10000}{2400} = I_{rms}$$

$$\Rightarrow \boxed{I_{rms} = 4.167 \text{ A}}$$

$$P = 2400 \times 4.167 \cos(36.86)$$

$$\boxed{P = 8000 \text{ W}}$$

$$Q = 10000 \sin(36.86)$$

$$\boxed{Q = 5998.61 \text{ VAR}}$$

$$Z = \frac{V_{rms}}{I_{rms}} \angle 36.86$$

$$\boxed{Z = 575.9539 \Omega}$$

13.11 $Q = 4.2 \text{ kVAR}$

$V_{rms} = 440 \text{ V}$

$f = 60 \text{ Hz}$

$I_{rms} = 12 \text{ A}$

$\omega = 376.8 \text{ rad/s}$

$PF = ?$

$Z = ?$

$V_{rms} I_{rms} \sin(\phi_v - \phi_i) = 4200$

$\phi_v - \phi_i = \arcsin(0.7954)$

$\phi_v - \phi_i = 37.35^\circ$

$Z = 36.67 e^{j37.35^\circ} \Omega$

$PF = 0.7954$

$Z = 36.67 e^{j52.69^\circ} \Omega$
 $PF = 0.6$

13.12 $I_{rms} = 22 \text{ A}$ $V_{rms} = 220 \text{ V}$

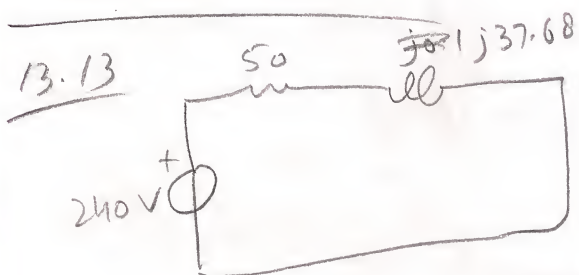
$PF = 0.9$ (inductive)

$Z = ?$ $\phi_v - \phi_i = 25.84^\circ$

$Z = 10 e^{j25.84^\circ} \Omega$

$PF = 0.8$ $\phi_v - \phi_i = 36.869^\circ$

$Z = 10 e^{j36.869^\circ} \Omega$



$f = 60 \text{ Hz}$ $\omega = 376.8 \text{ rad/s}$

$\hat{I} = \frac{240}{50 + j37.68} \times \frac{50 - j37.68}{50 - j37.68}$

$\hat{I} = \frac{12000 - 24j}{2500 + 1419.78j} = 1200$

$\hat{I} = 0.4799 - j0.009599$

$\hat{I} = 0.4799 e^{-j1.14588}$

$\hat{I} = 0.306 - j0.0061227$

$\hat{I} = 0.0936$

$\hat{I} = 3.06 - j2.307$

$\hat{I} = 3.83 \angle -37.01^\circ = 3.83 e^{-j37.01} \text{ A}$

$\hat{V} = (3.06 - j2.307)(50 + j37.68)$
 $\hat{V} = 3.83 e^{-j37.01} \cdot 62.608 e^{j37.01}$
 $= 239.78$

$\hat{V}_R = 3.83 e^{-j37.01} \times 50$

$\hat{V}_R = 191.5 e^{-j37.01} \text{ V}$

$\hat{V}_L = (3.83 e^{-j37.01})(j37.68)$

$\hat{V}_L = (3.06 - j2.307)(j37.68)$

$\hat{V}_L = 115.3008j + 86.927$

$\hat{V}_L = 144.397 \angle 52.986^\circ$

$\hat{V}_L = 144.397 e^{j52.986}$

$S = \hat{V} \cdot \hat{I}^*$
 $= 240 \cdot 3.83 e^{j37.01}$

$S = 919 e^{j37.01} \text{ VA}$



$$\omega = 376.8 \text{ rad/s}$$

$$S = (10 - j126) \text{ VA}$$

$$Z_{\text{eq}} = \frac{\frac{R}{j\omega C}}{j\omega R C + 1} = \frac{R}{1 + j\omega R C}$$

$$I_{\text{rms}} = \frac{440}{R} = \frac{440(1 + j\omega R C)}{R}$$

$$S = 440 \cdot \frac{440(1 - j\omega R C)}{R}$$

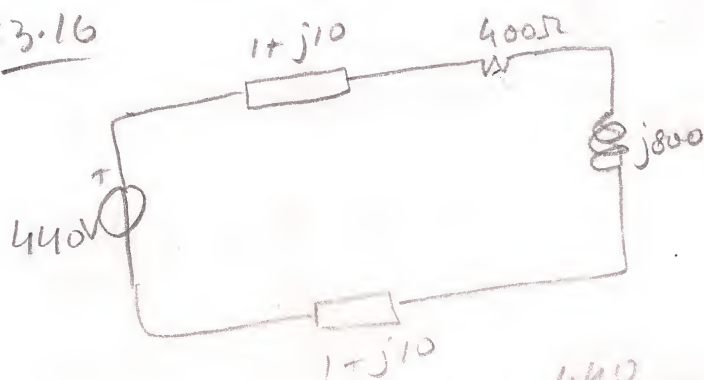
$$10 - j126 = \frac{193600}{R} - j\frac{72948480}{R} \text{ RC}$$

$$\frac{193600}{R} = 10 \Rightarrow R = 19360 \Omega$$

$$72948480 C = 126$$

$$C = 1.7 \mu\text{F}$$

$$13.16$$



$$\hat{I} = \frac{440}{2 + j20 + 400 + j800} = \frac{440}{402 + j820}$$

$$\hat{I} = \frac{176880 - 360800j}{834004}$$

$$\hat{I} = 0.212 - 0.4326j$$

$$\hat{I} = 0.48 e^{-j63.89} \text{ A}$$

$$\begin{aligned} b) S_L &= \hat{V}_L \hat{I}^* \\ &= \hat{I} (400 + j800) \hat{I}^* \\ &= 0.48 e^{-j63.89} \cdot 0.48 e^{j63.89} (400 + j800) \end{aligned}$$

$$S_L = 92.16 + 184.32j \text{ VA}$$

$$\begin{aligned} c) S_W &= \hat{V}_W \hat{I}^* \\ &= \hat{I} (2 + j20) 0.48 e^{j63.89} \\ &= 0.2304 (2 + j20) \end{aligned}$$

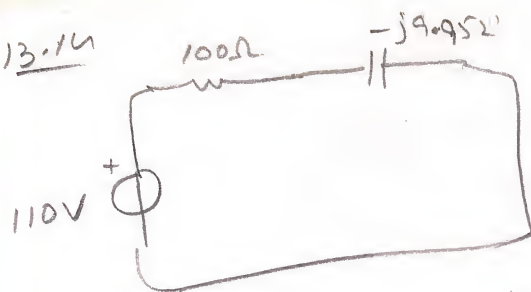
$$S_W = 0.4608 + j4.608$$

$$\begin{aligned} d) \eta &= \frac{P_L}{P_{\text{gen}}} \times 100 \\ &= \frac{206 \cos(63.43)}{0.48 \cos(-63.89)} \times 100 \\ &= \frac{92.1759}{0.2112} \times 100 \end{aligned}$$

$$\begin{aligned} \hat{V}_L &= (400 + j800) 0.48 e^{-j63.89} \\ &= 894.42 e^{j63.43} \cdot 0.48 e^{-j63.89} \\ \hat{V}_L &= 429.31 \angle 0^\circ \quad \hat{I} = 0.48 e^{-j63.89} \\ \hat{V}_{\text{gen}} &= 440 \angle 0^\circ \end{aligned}$$

$$\begin{aligned} P_L &= 206.07 \cos(63.89) \\ P_{\text{gen}} &= 440 \cos(63.89) \end{aligned}$$

13.14



$$\hat{I} = \frac{110}{100 - j9.952} \times \frac{100 + j9.952}{100 + j9.952}$$

$$= \frac{11000 + 1094.745j}{10000 + 10099.0423}$$

$$\hat{I} = 1.089 + 0.1084j$$

$$\hat{I} = 1.094 e^{j5.684} \text{ A}$$

13.14



$$Z_{eq} = \frac{-j995.2}{100 - j9.952} \times \frac{100 + j9.952}{100 + j9.952}$$

$$= \frac{-99520j + 9904.23j}{10099.0423}$$

$$Z_{eq} = 0.9807 - 9.85j$$

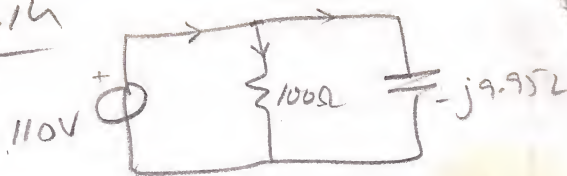
$$\hat{I} = \frac{110}{0.9807 - 9.85j} \times \frac{0.9807 + 9.85j}{0.9807 + 9.85j}$$

$$\hat{I} = \frac{107.877 + 1083.5j}{97.9842}$$

$$= 1.10096 + 11.0578j$$

$$\hat{I} = 11$$

13.14



$$\hat{V}_R = \frac{100}{100 - j9.952} \times 110$$

$$= \frac{11000(100 + j9.952)}{10099.0469}$$

$$= \frac{1100000 + 109472j}{10099.0468}$$

$$\hat{V}_R = 108.9211 + 10.839j$$

$$|\hat{V}_R| = 109.45$$

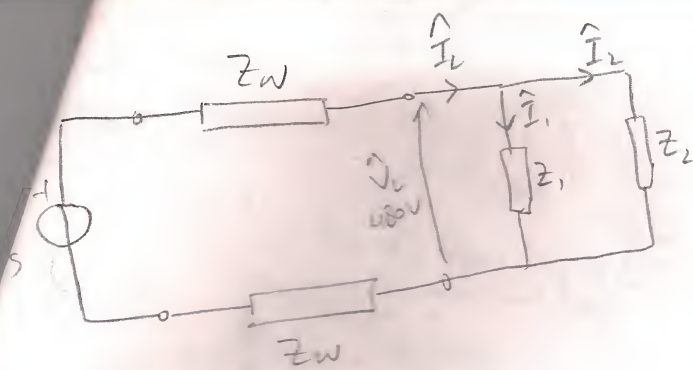
$$\hat{I}_R = \frac{110}{100} = 1.1 \text{ A} \checkmark$$

$$\hat{I}_C = \frac{110}{-j9.952} = +j11.05 \text{ A} \checkmark$$

$$S = 110(1.1 - j11.05) = vx \cdot i^*$$

$$S = 121 - 1215.83j$$

$$S = 1221.83 e^{-j84.31} \text{ VA}$$



$$\hat{V}_L = 480V$$

$$P(Z_1) = 10kW$$

$$(Z_1) PF = 0.8 \quad (\text{inductive})$$

$$P(Z_2) = 12kW$$

$$(Z_2) PF = 0.75 \quad (\text{inductive})$$

$$Z_W = 0.35 + j1.5 \Omega$$

$$S_S = ? \quad \hat{V}_S = ?$$

$$P_{Z_1} = \hat{V}_L \hat{I}_1 \cos(\phi_v - \phi_i)$$

$$(Z_1) \cos(\phi_v - \phi_i) = 0.8$$

$$\phi_v - \phi_i = 36.86^\circ \Rightarrow \phi_i = -36.86^\circ$$

$$\frac{10000}{480} = \hat{I}_1 \cos(36.86^\circ)$$

$$\hat{I}_1 = \frac{20.83}{0.8001} \Rightarrow \hat{I}_1 = 26.034A$$

$$P_{Z_2} = \hat{V}_L \hat{I}_2 \cos(\phi_v - \phi_i)$$

$$\frac{12000}{480} = \hat{I}_2 \cos(\phi_v - \phi_i)$$

$$(Z_2) \cos(\phi_v - \phi_i) = 0.75$$

$$\phi_v - \phi_i = 41.409^\circ \Rightarrow \phi_i = -41.409^\circ$$

$$\frac{25}{0.75} = \hat{I}_2 \Rightarrow \hat{I}_2 = 33.33A$$

$$\hat{I}_L = 26.034 e^{-j36.86^\circ} + 33.33 e^{-j41.409^\circ}$$

$$= 26.034 (\cos(36.86^\circ) - j \sin(36.86^\circ)) +$$

$$33.33 (\cos(-41.409^\circ) - j \sin(-41.409^\circ))$$

$$= 20.829 - j15.616 + 24.997 - j22.045$$

$$\hat{I}_L = 45.826 - j37.66 A$$

$$\hat{I}_L = 59.315 e^{-j39.41^\circ} A$$

$$Z_1 = \frac{480}{26.034 \angle -36.86^\circ} \Rightarrow Z_1 = 18.437 \angle 36.86^\circ$$

$$Z_1 = 14.75 + j11.059 \Omega$$

$$Z_2 = \frac{480}{33.33} \angle 41.409^\circ \Rightarrow Z_2 = 14.40 \angle 41.409^\circ$$

$$Z_2 = 10.8 + j9.52 \Omega$$

$$Z_{eq} = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{265.49 \angle 78.269^\circ}{14.75 + j11.059 + 10.8 + j9.52}$$

$$Z_{eq} = \frac{265.49 \angle 78.269^\circ}{25.55 + j20.579}$$

$$= \frac{265.49 \angle 78.269^\circ}{32.806 \angle 38.849^\circ}$$

$$Z_{eq} = 8.09 \angle 39.42^\circ$$

$$Z_{eq} = 6.249 + j5.137 \Omega$$

$$\hat{V}_s = (2Z_w + Z_{in}) \hat{I}_L$$

$$\hat{V}_s = (0.7 + 3j + 6.249 + j5.137) 59.315 e^{-j39.41}$$

$$\hat{V}_s = (6.949 + 8.137j) 59.315 e^{-j39.41}$$

$$\hat{V}_s = 10.7 e^{j49.502} \cdot 59.315 e^{-j39.41}$$

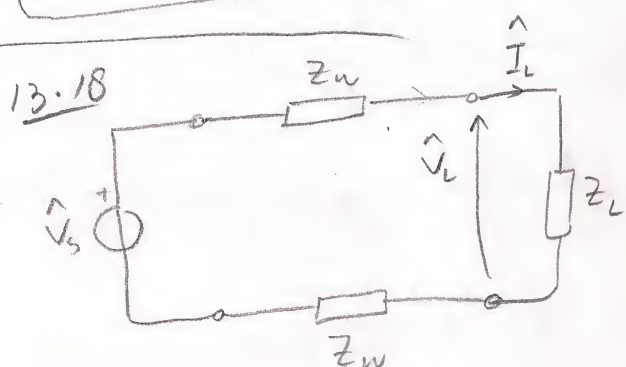
$$\hat{V}_s = 634.67 e^{j10.09}$$

$$\hat{V}_s = 624.85 + 111.19j$$

$$S_s = \hat{V}_s \cdot \hat{I}_L^* = 634.67 e^{j10.09} \cdot 59.315 e^{j39.41}$$

$$S_s = 37645.45 e^{j49.5}$$

$$S_s = 24448.76 + 28625.82j$$



$$A(Z_L) = 2.5 \text{ kVA}$$

$$PF = 0.9 \quad (\phi_v - \phi_i) = 25.84^\circ$$

$$\phi_v = 25.84 - 28.35$$

$$\phi_v = -2.51^\circ$$

$$A(\hat{V}_s) = 2.65 \text{ kVA}$$

$$PF = 0.88$$

$$\phi_v - \phi_i = 28.35^\circ$$

$$\phi_i = -28.35^\circ$$

$$Z_L = ?$$

$$\hat{I}_L = ?$$

$$Z_w = ?$$

$$A(\hat{V}_s) = \hat{V}_s \hat{I}_L$$

$$\hat{I}_L = \frac{2650}{2400} \Rightarrow \hat{I}_L = 1.083$$

$$\hat{I}_L = 1.104 \angle -28.35^\circ$$

$$\hat{I}_L = 1.104 e^{-j28.35^\circ} \text{ A}$$

$$A(Z_L) = \hat{V}_L \hat{I}_L$$

$$\hat{V}_L = \frac{2500}{1.104}$$

$$\hat{V}_L = 2264.49 e^{-j2.51^\circ} \text{ V}$$

$$Z_L = \frac{2264.49 \angle -2.51^\circ}{1.104 \angle -28.35^\circ}$$

$$Z_L = 2051 \angle 25.84^\circ$$

$$Z_L = 2051 e^{j25.84^\circ} \Omega$$

$$\hat{V}_s = \hat{I}_L (2Z_w + Z_L)$$

$$\frac{2400 \angle 0}{1.104 \angle -28.35} = 2Z_w + 2051 e^{j25.84}$$

$$2173.9 \angle 28.35 - 2051 \angle 25.84 = 2Z_w$$

$$Z_w = 1086.95 \angle 28.35 - 1025.5 \angle 25.84$$

$$Z_w = 956.58 + j516.145 - 922.96 - j446.97$$

$$Z_w = 33.62 + j69.171$$

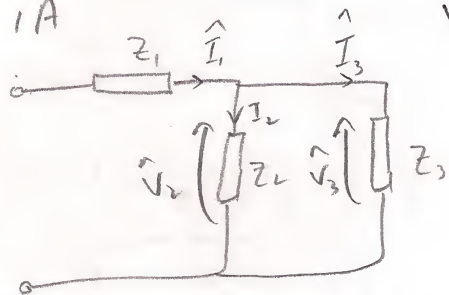
$$Z_w = 76.9 \angle 64.07^\circ$$

$$P_1 = 0 \text{ W}, Q_1 = -10 \text{ VAR}$$

$$P_2 = 20 \text{ W}, Q_2 = 10 \text{ VAR}$$

$$P_3 = 10 \text{ W}, Q_3 = 10 \text{ VAR}$$

$$|\hat{I}_3| = 1 \text{ A}$$



$$|13.201$$

$$P_3 = \hat{V}_3 \hat{I}_3 \cos(\phi_v - \phi_i)$$

$$\frac{10}{10} = \cos(\phi_v - \phi_i)$$

$$\phi_v - \phi_i = 90^\circ$$

$$\phi_v = 90^\circ$$

$$\hat{V}_3 = \hat{V}_2$$

$$\hat{Z}_3 = \frac{\hat{V}_3}{\hat{I}_3}$$

$$\hat{Z}_3 = 10 e^{j90^\circ} \Rightarrow \hat{Z}_3 = 10 + j10 \Omega$$

$$\hat{Z}_3 = 10 + j10 \Omega$$

$$|\hat{V}_3| = \sqrt{200}$$

$$\hat{V}_3 = \frac{V_r}{\sqrt{2}} = \frac{10\sqrt{2}}{\sqrt{2}} = 10 \text{ V}$$

$$P_2 + jQ_2 = \hat{V}_2 \hat{I}_2^*$$

$$\frac{20 + j10}{10 + j10} = \hat{I}_2^*$$

$$\hat{I}_2^* = \frac{(20 + j10)(10 - j10)}{200}$$

$$\hat{I}_2^* = \frac{200 + 100 + 100j - j200}{200}$$

$$\hat{I}_2^* = 1.5 - 0.5j$$

$$\hat{I}_2 = 1.5 + 0.5j$$

$$\hat{Z}_2 = \frac{10 + j10}{1.5 + 0.5j} \times \frac{1.5 - 0.5j}{1.5 - 0.5j}$$

$$= \frac{15 - 5j + 15j + 5}{2.5} = \frac{20 + 10j}{2.5}$$

$$\hat{Z}_2 = 8 + 4j$$

$$\hat{I}_1 = 1.5 + 0.5j + 1$$

$$\hat{I}_1 = 2.5 + 0.5j$$

$$P_1 + jQ_1 = \hat{V}_1 \hat{I}_1^*$$

$$\frac{0 - j10}{2.5 + 0.5j} = \hat{V}_1 \Rightarrow \hat{V}_1 = \frac{-j10}{2.5 + 0.5j} \times \frac{2.5 - 0.5j}{2.5 - 0.5j}$$

$$\hat{V}_1 = \frac{-25j - 5}{6.5} = -0.769 - 3.84j \text{ V}$$

$$\hat{V}_1 = \frac{5 - 25j}{6.5}$$

$$\hat{V}_1 = 0.769 - 3.84j \text{ V}$$

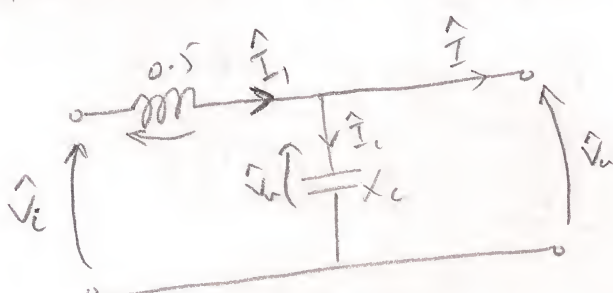
$$\hat{Z}_1 = \frac{0.769 - 3.84j}{2.5 + 0.5j} \times \frac{2.5 - 0.5j}{2.5 - 0.5j} = \frac{1.923 - 0.3845j - 9.6j - 1.92}{6.5}$$

$$\hat{Z}_1 = -j1.53 \Omega$$

$$13.21 \times C = ?$$

$$|\hat{V}_1| = 120 \text{ V}, |\hat{V}_2| = 100 \text{ V}, |\hat{I}| = 10 \text{ A}$$

$$Q = 600 \text{ VAR}, X_L = 0.5 \Omega$$



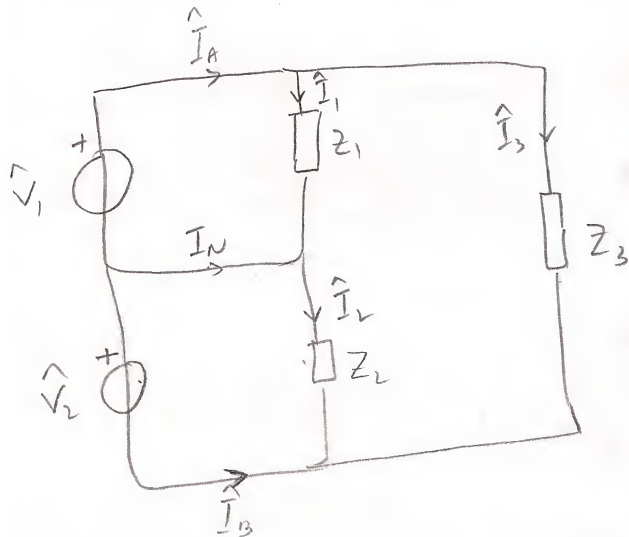
13.18

$$S_1 = (1250 + j250) \text{ VA}$$

$$S_2 = (800 + j400) \text{ VA}$$

$$S_3 = (2000 + j0) \text{ VA}$$

$$\hat{V}_1 = \hat{V}_2 = 110 \angle 0^\circ \text{ V}$$



$$S_1 = \hat{V}_1 \cdot \hat{I}_1^*$$

$$\frac{1250 + j250}{110} = \hat{I}_1^*$$

$$\hat{I}_1^* = 11.36 + 2.27j$$

$$\hat{I}_1 = 11.36 - 2.27j$$

$$S_2 = \hat{V}_2 \cdot \hat{I}_2^*$$

$$\frac{800 + j400}{110} = \hat{I}_2^*$$

$$\hat{I}_2^* = 7.27 + j3.636$$

$$\hat{I}_2 = 7.27 - j3.636$$

$$S_3 = (\hat{V}_1 + \hat{V}_2) \hat{I}_3^*$$

$$\hat{I}_3^* = \frac{2000 + j0}{220}$$

$$\hat{I}_3 = 9.09 - j0$$

$$\hat{I}_A = \hat{I}_1 + \hat{I}_3$$

$$= 11.36 - 2.27j + 9.09 - j0$$

$$\hat{I}_A = 20.45 - 2.27j \text{ A}$$

$$\hat{I}_N = \hat{I}_2 - \hat{I}_1$$

$$= 7.27 - j3.636 - 11.36 + 2.27j$$

$$\hat{I}_N = -4.09 - 1.366j \text{ A}$$

$$\hat{I}_B = -\hat{I}_2 - \hat{I}_3$$

$$= -7.27 + j3.636 - 9.09$$

$$\hat{I}_B = \frac{-16.36}{-16.36} + j3.636 \text{ A}$$

$$S_{V_1} = \hat{V}_1 \cdot \hat{I}_A^*$$

$$= 110 (20.45 + 2.27j)$$

$$S_{V_1} = 2249 + 249.7j \text{ VA}$$

$$S_{V_2} = -\hat{V}_2 \cdot \hat{I}_B^*$$

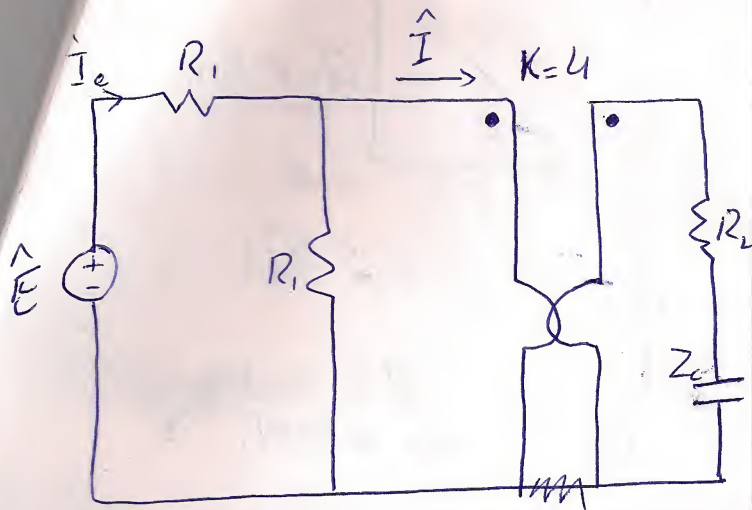
$$= 110 (16.36 + j3.636)$$

$$S_{V_2} = 1799.6 + 399.96j$$

$$e(t) = 10 \cos(\omega t) \text{ V}$$

19-12-2014

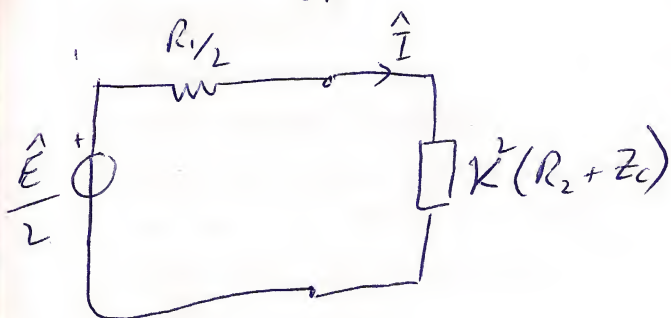
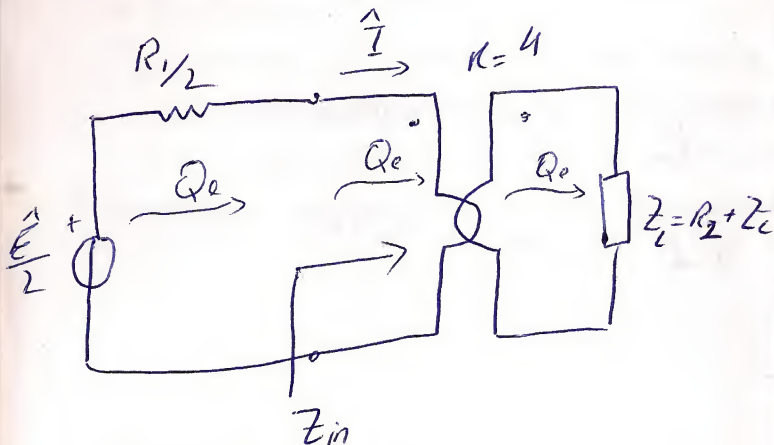
We will calculate power in capacitance only



$$R_1 = 10 \Omega, R_2 = 5 \Omega, Z_c = -j20 \Omega$$

$$\hat{I}, Q = ?$$

$$I = \frac{E}{R_1} = \frac{10}{2}$$



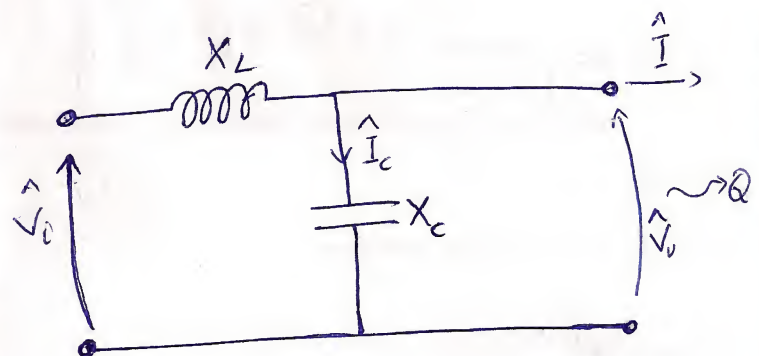
$$\hat{I} = \frac{\hat{E}/2}{R_1/2 + K^2(R_2 + Z_c)}$$

13.21

Evaluate X_c

$$|\hat{V}_i| = 120 \text{ V}, |\hat{V}_u| = 100 \text{ V}, |\hat{I}| = 10 \text{ A},$$

$$Q = 600 \text{ VA}, X_L = 0.5 \Omega$$



$$Q = |\hat{V}_u| |\hat{I}| \sin \phi$$

$$\phi = \arcsin \frac{Q}{|\hat{V}_u| |\hat{I}|} = \arcsin \frac{600}{1000}$$

$$= \arcsin 0.6$$

$$\phi = 37^\circ \checkmark$$

Phase shift

$$\phi = \angle \hat{V}_u - \angle \hat{I} \Rightarrow \angle \hat{I} = \angle \hat{V}_u - \phi = -37^\circ$$

$$\angle \hat{V}_u = 0 \text{ (reference phase)}$$

$$\hat{I} = 10 e^{-j37^\circ} = 10 (\cos 37^\circ - j \sin 37^\circ)$$

$$\hat{V}_u = 100 \text{ V} \Rightarrow \hat{I} = (8 - j6) \text{ A}$$

$$\hat{I} = 10(\cos \varphi - j \frac{\sin \varphi}{0.6})$$

$$\hat{I}_c = \frac{\hat{V}_0}{jX_c} = \frac{100}{jX_c} \quad (\text{reactive}), X_c < 0$$

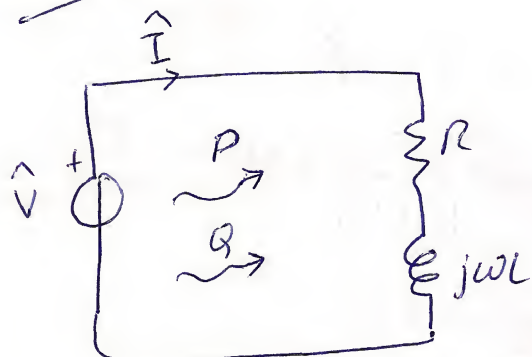
$$\hat{V}_c = \hat{V}_0 + j0.5(\hat{I} + \hat{I}_c)$$

$$|\hat{V}_c| = \left| 100 + j0.5 \left(8 - 6j + \frac{100}{jX_c} \right) \right|$$

$$120 = \left| 100 + j0.5 \left(8 - 6j + \frac{100}{jX_c} \right) \right|$$

$$X_c = ?$$

Example



Find P, Q As functions of
P.F. = $\cos \varphi$

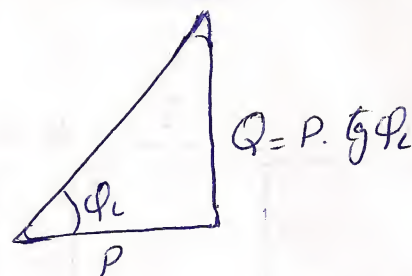
$$\varphi = \angle Z_L = \arctan \left(\frac{\omega L}{R} \right)$$

$$Z_L = R + j\omega L$$

$$P = |\hat{V}| |\hat{I}| \cos \varphi$$

$$Q = |\hat{V}| |\hat{I}| \sin \varphi$$

$$|\hat{I}| = \frac{P}{|\hat{V}| \cos \varphi}$$



Fix P , Fix $|\hat{V}|$

$$\varphi_L \uparrow \Rightarrow \cos \varphi_L \downarrow$$

$$|\hat{I}| \uparrow$$

$$\varphi_L < \varphi_{max}$$

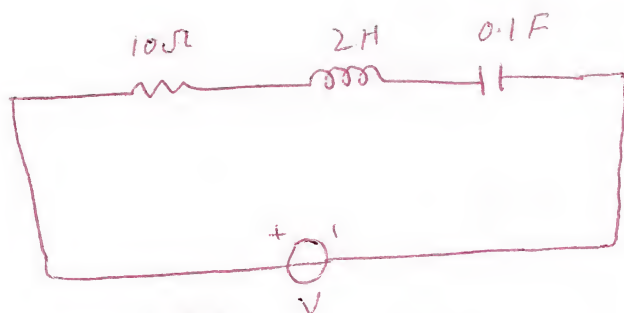
$$\cos \varphi_L > PF_{min}$$

$$\varphi_{in} = \angle Z_{in} \quad \text{Fixed}$$

Total power factor correction

$$\varphi_{in} = 0^\circ$$

13.05



$$V_{rms} = 10 \text{ V}$$

$$\omega = 1 \text{ rad/s}$$

Average Power supplied by source?

Solution

Suppose $\angle V = 0^\circ \Rightarrow \hat{V} = 10\sqrt{2} \text{ V}$

So

$$Z_{eq} = 10 + 2j - \frac{j}{0.1}$$

$$= 10 - 8j$$

$$\hat{I} = \frac{\hat{V}}{10 - 8j} = 1.10 \angle 38.66^\circ$$

$$\text{Average Power} = \frac{1}{2} |\hat{V}| |\hat{I}| \cos(\theta_v - \theta_i) \text{ W}$$

$$= \frac{1}{2} (10\sqrt{2}) (1.10) \cos(38.66^\circ)$$

$$= \cancel{5.52} \Rightarrow 6.07 \text{ W}$$

{ ANSWER NOT VERIFIED!
BUT LETS TRY TO FIGURE-OUT
THE PROBLEM. 😊

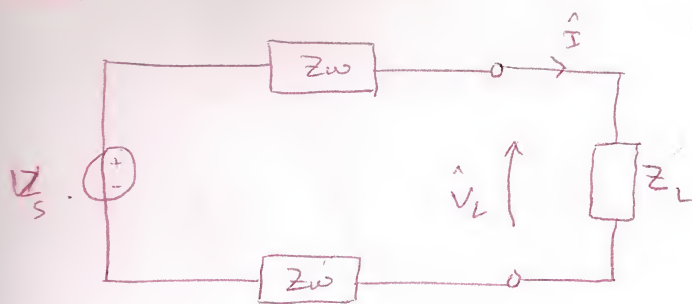
if we take $|\hat{V}| = 10 \text{ V}$

$$\hat{I} = 0.788 \angle 38.66^\circ$$

$$P = 3.048 \text{ W} \Rightarrow$$

{ ANSWER **VERIFIED** if we
ignore (rms) FACTOR and TREAT
NORMALLY. YES (PROF. VERIFIED)

13.18



(My Answer is Right)

$$A_{Z_L} = 2.5 \text{ KVA} \Rightarrow \text{P.F.} = 0.9$$

$$\theta_1 = 25.84^\circ$$

$$V_s(\text{rms}) = 2400 \text{ V} \Rightarrow \text{P.F.} = 0.88, A_s = 2.65 \text{ KVA} \quad \theta_2 = 28.36^\circ$$

$$\hat{I}_L, \hat{Z}_L, \hat{Z}_w \Rightarrow ?$$

Solution

$$i) |\hat{V}_s| = \sqrt{2} \times 2400 = 3394.11 \text{ V}$$

$$A_s = \frac{1}{2} |\hat{V}_s| |\hat{I}_s|$$

$$|\hat{I}_s| = \frac{2 \times 2.65 \times 10^3}{3394.11} = 1.56 \text{ A}$$

ERROR

$$G-A = 2.21 e^{-j28.36^\circ}$$

By taking $\angle V_s = 0^\circ$

$$\angle V_s - \angle I_s = 28.36, \angle I_s = -28.36 \Rightarrow \hat{I}_s = (1.56 e^{-j28.36^\circ}) \text{ A}$$

2) we know

$$S = A e^{j\theta} \text{ VA}$$

So

$$S_s = 2.65 e^{j28.36^\circ} \text{ KVA}$$

$$S_L = 2.5 e^{j25.84^\circ} \text{ KVA}$$

where

$$S_w + S_z = S_s$$

$$S_z \text{ (Complex Power of } 2Z_w) \\ = 2.65 e^{j28.36^\circ} - 2.5 e^{j25.84^\circ} \text{ KVA}$$

$$KVA (2.5 e^{j25.84^\circ}) = \frac{1}{2} (Z_L) |I_s|^2$$

$$Z_L = \frac{5 \angle 25.84^\circ}{2.43} \text{ K } \Omega$$

$$Z_L = 2.05 \angle 25.84^\circ \text{ K } \Omega$$

ERROR

$$G-A = 1.02 \angle 25.84^\circ \text{ K } \Omega$$

$$iii) KVA \left(2.65e^{j28.35^\circ} - 2.5e^{j258.4^\circ} \right) = \frac{1}{2} Z_w' |I_s|^2$$

$$K\Omega \left(\frac{2 \cdot (2.33 + j1.259 - 2.25 - 1.09j)}{|I_s|^2} \right) = Z_w'$$

$$K\Omega \left(\frac{0.16 + 0.39j}{|I_s|^2} \right) = Z_w'$$

$$Z_w' = \left(\frac{0.35}{|I_s|^2} \angle 63.43^\circ \right) K\Omega$$

$$Z_w' = 143.81 e^{j63.43^\circ}$$

$$Z_w = \frac{143.81}{2} e^{j63.43^\circ} = (71.9 e^{j63.43^\circ}) \Omega$$

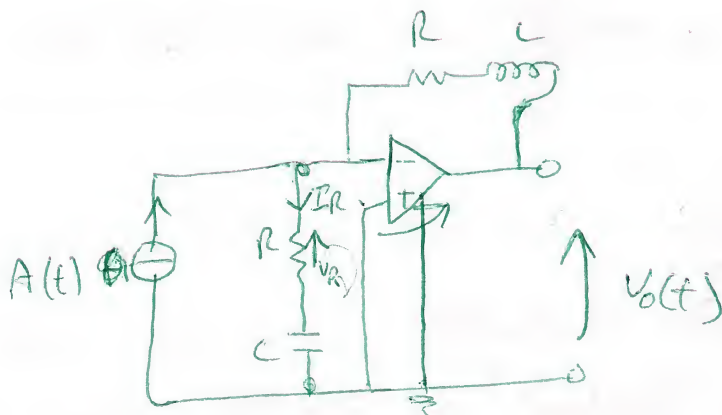
$$S = Ae^{j\phi}$$

$$S = \frac{1}{2} I Z (I)^*$$

$$= \frac{1}{2} |I|^2 Z$$

ERROR

$$G-A = 38.45 e^{j64.11^\circ} \Omega$$

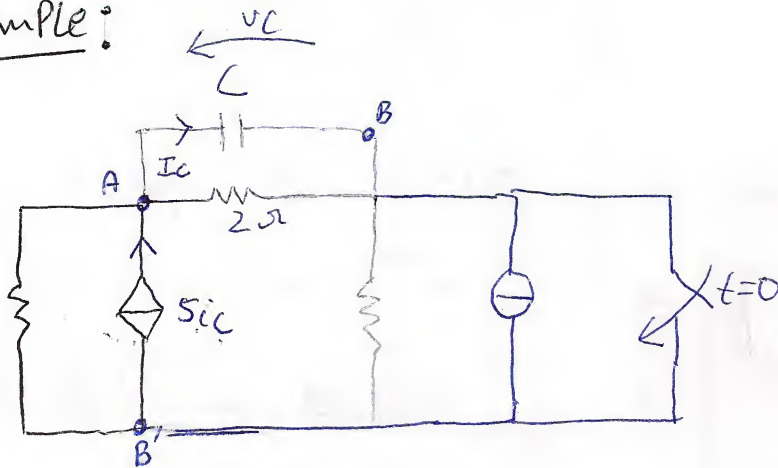


$$V_R = ? , I_R = ?$$



// WHAT happened to dependent source when we find initial conditions? Any specific/special thing to keep in mind for that?

Example:



i) For $t \rightarrow 0^-$ or $t < 0$, $i_c = 0$ AB open \Rightarrow AB' also open? ✓

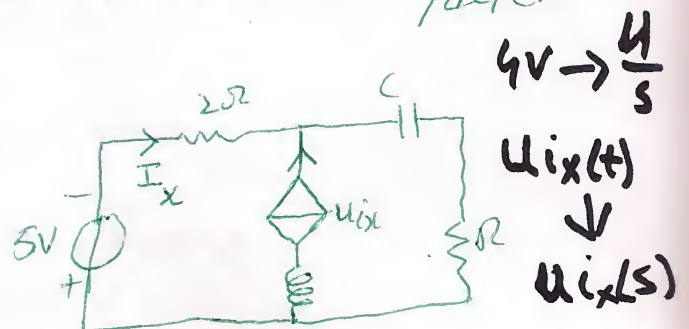
OR

We just assume AB' open in case of current source and short for voltage source. (Dependent ones) ✗

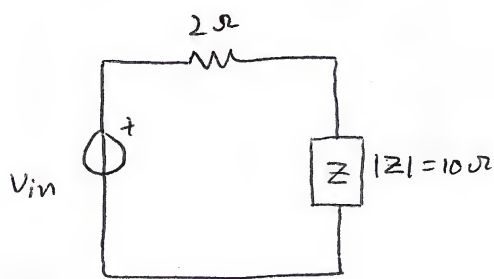
ii) WHAT happened when instead of D. current source we have a dependent voltage source. ($\uparrow 2v_c$)

WE HAVE TO SOLVE IT AS "CH#4"

// TRANSFORMING from Real circuit (time domain) to Symbolic circuit (Laplace or frequency domain), what happens to constant value or independent/dependent voltage/current source?



13.9



$$P_Z = 6W, P_{in} = 8W$$

$$P_{in} = P_R + P_Z$$

$$P_R = 8 - 6 = 2W$$

$$P_R = \frac{1}{2} |\hat{I}|^2 R$$

$$a) |\hat{I}| = \sqrt{\frac{2 \times 2}{2}} = \sqrt{2} = 1.41 A$$

b) Power Factor of Z,

$$P_Z = \frac{1}{2} |V_Z| |I_Z| \overbrace{\cos \theta}^{\text{PF of Z}} \quad \left\{ |V_Z| = |I_Z| |Z| \right.$$

$$6 = \frac{1}{2} |I_Z|^2 |Z| \cos \theta$$

$$\cos \theta = \frac{12}{10 \div (1.41)^2}$$

$$P.F = 0.6$$

c) P.F of Load Seen From Source Terminal:-

Two - Methods

$$P_{in} = \frac{1}{2} |\hat{V}_{in}| |\hat{I}| \cos \theta$$

$$\cos \theta = \frac{2 \times P_{in}}{|\hat{I}|^2 |Z_{eq}|}$$

$$P.F = \frac{16}{(1.41)^2 (8.52)} = 0.71$$

$$\left\{ \begin{aligned} |V_{in}| &= |\hat{I}| |2 + 10j| \\ \angle Z &= \cos^{-1} 0.6 \\ \angle Z &= 53.13^\circ \end{aligned} \right.$$

$$Z_{eq} = R + Z$$

$$= 2 + 10e^{j53.13} = (8 + j8) \Omega$$

$$\theta = \tan^{-1} \frac{8}{8} = 45^\circ$$

$$P.F = \cos \theta = 0.707$$

Let $\angle I = 0$, Then $\angle Z_{eq} = \angle V - \angle I$
 $\angle V = 45^\circ$

D)

$$V_{in}(t) = \hat{I} \cdot \hat{Z}_{eq} = (15.45 \angle 45^\circ) V$$

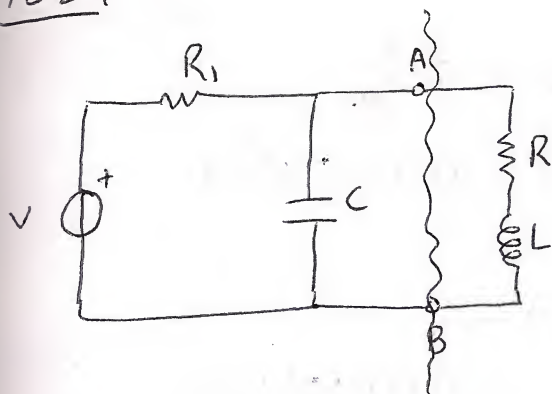
$$E) V_Z(t) = \hat{I} Z = (1.41)(10 \angle 53.13^\circ) = (14.1 \angle 53.13^\circ) V$$

if we let $\angle V = 0$, then $\angle I = -45^\circ$

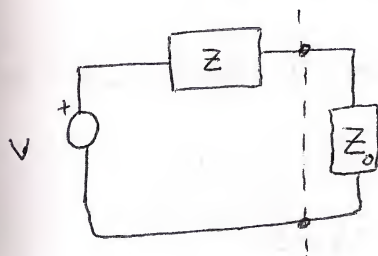
$$\left\{ \begin{aligned} V_{in}(t) &= 16 \cos(\omega t) V \\ V_Z(t) &= 14.1 e^{j(53.13 - 45)} \\ &= 14.1 e^{j8.13^\circ} \\ V_Z(t) &= 14.1 \cos(\omega t + 8.13^\circ) V \end{aligned} \right.$$

Verified ✓

13.24



For MAX-Power output



$$Z_C = \frac{-j10^6}{40 \times 2\pi \times 10^3}$$

$$= -j39.74 \Omega$$

$$Z_0 = Z^* \Rightarrow Z_0 = R + j\omega L$$

$$\text{So } Z = R_1 \parallel Z_C$$

$$= \frac{(3)(-j39.7)}{3 - j39.7}$$

$$Z_0 = Z^* = \frac{j12}{3 + j3.47} \times \frac{3 - j3.97}{3 - j3.97}$$

$$R = \frac{48}{25} = 1.92 \Omega \checkmark$$

$$j\omega L = \frac{36j}{25} = 1.44$$

$$L = \frac{1.44}{2\pi f} = 0.229 \text{ mH} \checkmark$$

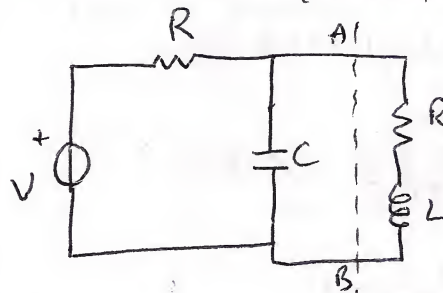
$$V_{\text{eff}} = V \left(\frac{Z_C}{R + Z_C} \right)$$

$$\left| \frac{R + Z_C}{Z_C} \right| |5.52 \angle 0| = |\hat{V}|$$

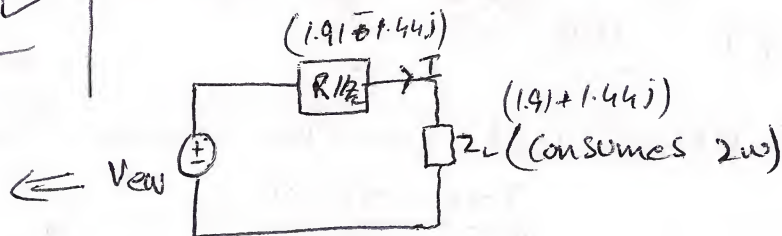
$$|\hat{V}| = 6.92 \text{ V}$$

Now we know that under Power matching Conditions Z_0 consumes 2w active Power. and we have to find $|\hat{V}|$.

~~$Z_{\text{eq}} = 2 \parallel R_2 \parallel Z_0 \parallel Z_0 \parallel Z_0$~~



Thevenin equivalent says that (AB)

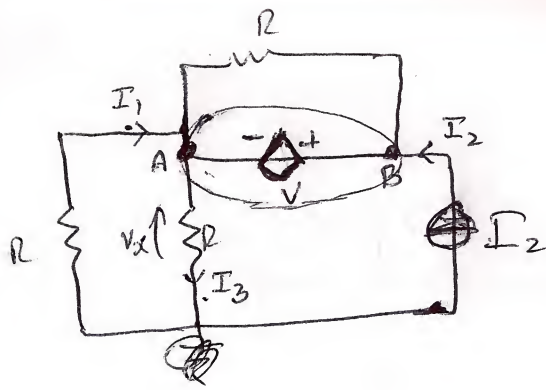


$$\hat{I} = \frac{V_{\text{eff}}}{3.82}$$

$$P_s = \frac{1}{2} \frac{V_{\text{eff}}}{3.82} \cdot (1.91 + j1.44j) \cdot \frac{(V_{\text{eff}})^*}{3.82}$$

$$2 = \frac{1.91 |V_{\text{eff}}|^2}{2 \times 3.82 \times 3.82} \Rightarrow V_{\text{eff}} = 5.528 \text{ V}$$

$\frac{P_2}{P_3}$



$$I_1 - I_3 + I_2 = 0$$

Taking AB AS Supernode,
what will be KCL:-

we will ignore the Branch current
which is Parallel to SUPER-NODE

- i) For Req of a CKT (Ch#9) Prefer Test source Method.
ii) You will find this Ratio in Exam (ALMOST)

CH	no. Q
1-4	2
6	1

9 } 1 Short (MAY Be one Big Question ALSO)
10 } 1 Short

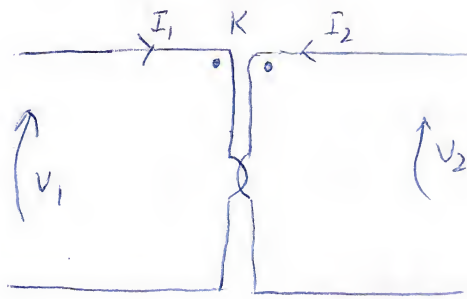
11 } Big Question
12 } 1 Short (MAY Be Big ALSO)

13 } 1 —————

14 } 1 Short —————

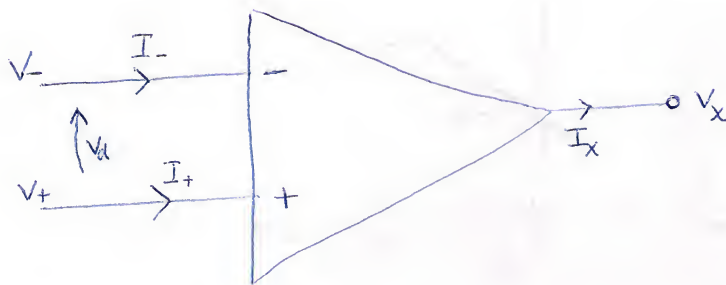
You must have complete grip on (KCL, KVL, MILLMAN
Thevenin/NORTON)

TRANSFORMER:-



$$\left. \begin{aligned} V_1 &= K V_2 \quad \text{--- (i)} \\ I_1 &= -\frac{1}{K} I_2 \quad \text{--- (ii)} \end{aligned} \right\} \underline{\text{ALWAYS}}$$

AMPLIFIER:-



$$V_- = V_+, \text{ so } V_d = 0 \quad \bigg| \quad \text{ALSO } I_- = I_+ = 0$$

BUT $\Rightarrow V_x, I_x$ ARE UNKNOWN VARIABLES,
